UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 11 June 2007 1.30 to 4.30

PAPER 26

SET THEORY AND LOGIC

Questions in Part One are worth one credit.

Questions in Part Two are worth two credits.

Six credits equate to full marks.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

PART ONE

1 What is an ultralimit? Prove that if $\mathcal{A}$ and $\mathcal{B}$ are elementarily equivalent, then they have isomorphic ultralimits. You may assume Łoś's theorem.

2

(i) Show that there is no order-preserving embedding from chains-in-P to P, where P is a poset.

(ii) Show that the relation $\mathcal{P}(x \cap y) \subseteq y$ is wellfounded. (You may not use foundation.)

3 Let κ be supercompact: show that Σ_2 sentences generalise downward to V_κ.

4 State and prove the Gale–Stewart theorem, and the strengthened version for games where the payoff set is a countable intersection of open sets.

5 State and prove Łoś's theorem. Use it to give an ultraproduct proof that if T is a theory all of whose finite fragments have models then T has a model.

6 State and prove the Ehrenfeucht–Mostowski theorem. You may assume Łoś's theorem or Ramsey's theorem.

PART TWO

7 What is a measurable cardinal? An elementary embedding? Can there be an elementary embedding from the universe into itself?

8 Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

9 State and prove Kruskal's theorem on wellquasiorders of trees, and deduce Friedman's Finite Form from it.

10 Let A be an arbitrary set; give it the discrete topology, and A^ω the product topology. Show that games played over A whose payoff is Borel have winning strategies.

END OF PAPER