MATHEMATICAL TRIPOS Part III

Monday 9 June 2008 9.00 to 12.00

PAPER 72

GALAXIES

Attempt no more than **THREE** *questions.*

There are **FOUR** *questions in total.*

The questions carry equal weight.

This is an **OPEN BOOK** *examination.*
Candidates may bring handwritten notes and lecture handouts into the examination.

STATIONERY REQUIREMENTS	***SPECIAL REQUIREMENTS***
Cover sheet *Treasury tag* *Script paper*	*None*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Consider a galaxy that forms initially from a completely gasous state with mass $M(0)$, and initial metal abundance $Z = 0$. The system initially evolves as a closed system, and forms stars at a rate ψ that scales linearly with the mass of available gas. The newly formed stars return a fraction F of their mass back to the gas, enriched in metals with yield Y. Y is defined as the mass of metals produced by a stellar generation normalized to the mass locked up in newly formed stars.

(i) Derive an expression for the time evolution of the gas mass of the system, and for the time evolution of the star formation rate.

(ii) Derive an expression for the time evolution of the metal abundance Z, in the limit $Z \ll 1$.

(iii) Derive an expression for the metallicity distribution of the stars. I.e., the number of stars of metallicity Z per unit interval dZ. HINT: The relation is independent of the time evolution of the system.

(iv) Show that the metallicity of gas Z can be expressed independent of time, in terms of the yield and gas fraction $\mu = M_{\rm gas}/M(0)$ as:

$$Z = Y \ln\left(\frac{1}{\mu}\right). \qquad (1)$$

(v) How well do these simple predictions of the evolution and metallicity properties of stellar populations agree with the observed stellar populations in the Milky Way? What factors are thought to account for any discrepancies?

2 Consider a galaxy composed of stars, with a constant rotation velocity about the z axis of v, and anisotropic velocity dispersion characterized by a parameter $\sqrt{1-\beta} = \sigma_z/\sigma_x$. (For simplicity assume $\sigma_x = \sigma_y$.)

(i) Show that the intrinsic flattening of the galaxy ($e \equiv 1 - b/a$, where a and b are the major and minor axis dimensions, respectively) is related to its kinematical properties by the relation:

$$\frac{v}{\sigma_x} = \sqrt{\frac{e-\beta}{1-e}} \,. \tag{2}$$

(ii) Draw a schematic diagram plotting v/σ as a function of ellipticity e, indicating the appropriate range of observed values on each axis. Sketch the behaviour of the relation above for different values of β, and indicate the regions of the diagram occupied by giant elliptical galaxies, low-luminosity elliptical galaxies, and spiral galaxy bulges. Briefly describe (in an accompanying caption or paragraph) the differences in the observed loci for each of these types of objects.

(iii) Consider a disc galaxy composed of a gas disc rotating at the circular velocity V_c and negligible velocity dispersion, and a stellar disc with isotropic velocity dispersion σ (with $\sigma < V_c$). Derive an expression for the rotational velocity of the stars, in terms of V_c and σ. How valid are these approximations relative to the actual kinematics of stars in discs?

(iv) Draw another schematic diagram showing the dependence on radius of the rotational velocity of (a) the gas in a galaxy that is similar in type and mass to the Milky Way; (b) the rotation velocity of the stars in the same disc, using the results from above with realistic values for the kinematical parameters. As before indicate approximate representative numerical scales on each axis.

3 Consider a dwarf companion galaxy in orbit around a giant parent galaxy. Each is a spheroidal galaxy, with a distribution of stars approximated by an isothermal sphere:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}\,, \tag{3}$$

where σ is the (constant) velocity dispersion and r the radius from the centre of each galaxy.

(i) Derive an expression for the total mass enclosed within radius R for this profile, and the circular velocity V_c for test particles orbiting at radius R. How does the latter compare to the observed rotation curves of galaxies? In what ways is the isothermal profile an *unrealistic* description of real galaxies?

(ii) The density profile of the dwarf galaxy is truncated by the gravitational field of the giant galaxy. Derive an expression for the truncation radius in terms of the velocity dispersions and radial separation of the two galaxies.

(iii) Using the result above, derive an expression for the mass of the dwarf spheroidal galaxy, again in terms of its velocity dispersion and separation from the parent galaxy.

(iv) Derive the value of the radius and mass of the dwarf galaxy assuming its velocity dispersion is $10\,\mathrm{km\,s^{-1}}$, and it is orbiting at a distance of 50kpc from a giant galaxy with velocity dispersion $300\,\mathrm{km\,s^{-1}}$. Calculations to one significant figure are sufficient (here and in what follows).

(v) Suppose that the optical (e.g., blue) luminosity of the giant galaxy is $10^{11}\mathrm{L}_\odot$, which is a typical observed value for a system with $\sigma = 300\,\mathrm{km\,s^{-1}}$. Estimate the luminosity of the dwarf galaxy (in solar units), if it lies on the main fundamental plane for elliptical galaxies (i.e., Faber-Jackson relation). Derive the mass/light ratio of the dwarf galaxy, and compare it with the values observed for (a) typical giant elliptical galaxies, and (b) dwarf spheroidal galaxies in the local Universe.

(vi) Compare the mass/light ratio of the dwarf galaxy to that of the giant parent galaxy (within a 50kpc radius). How can the two values be so different, when they lie on the same Faber-Jackson relation? (Recall that when we derived the relation we assumed that all elliptical galaxies have the same M/L ratio.)

Useful Information:

$$\begin{aligned}
\mathrm{G} &= 7\times10^{-8}\mathrm{cm^3g^{-1}s^{-2}} \;=\; 7\times10^{-11}\mathrm{m^3kg^{-1}s^{-2}}\\
\mathrm{M}_\odot &= 2\times10^{33}\mathrm{g} \;=\; 2\times10^{30}\mathrm{kg}\\
1\ \mathrm{pc} &= 3\times10^{18}\mathrm{cm} \;=\; 3\times10^{16}\mathrm{m}\\
1\ \mathrm{yr} &= 3\times10^{7}\mathrm{s}
\end{aligned}$$

4 An underlying theme throughout the course was the comparison of observed properties of galaxies with expectations from the current ΛCDM hierarchical model, which integrates a picture for the growth of galaxies with the buildup of the large scale structure of the Universe itself.

(i) Describe, in roughly a paragraph each, 5 general observations of galaxies (that can include the Milky Way), which provide empirical support for the hierarchical assembly of galaxies. Exclude the obvious one, i.e. the observation of occasional instances of galaxies merging at the present time. In each case explain why the same observations are not as readily understood in the traditional formation/evolution model, which proposed that the formation of galaxies was largely completed in single rapid collapse events more than 10 Gyr ago.

(ii) Describe in the same way 3 general observations of galaxies (that can include the Milky Way) which cannot be reproduced by current models or simulations of galaxies within the ΛCDM framework. For each case explain whether these observations can be understood more readily in the old instantaneous collapse picture, and if so how. You should include at least one observation that is more consistent with the rapid collapse theory.

END OF PAPER