UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 4 June 2002 1.30 to 3.30

PAPER 75

EARLY UNIVERSE COSMOLOGY

Attempt ***TWO*** *questions*

There are ***four*** *questions in total*

The questions carry equal weight

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (a) In either Newtonian or synchronous gauge, write down the Sachs-Wolfe equation describing the linear sources of microwave anisotropies. Explain the physical significance of each term and describe how each changes with angle, θ, for scale invariant primordial fluctuations.

(b) Some primordial theories, such as inflation, predict coherent, adiabatic perturbations which give rise to so-called 'Doppler peaks' in the CMB power spectrum. Describe the origin of the first three peaks and calculate the spacing between the peaks.

(c) Describe two effects which suppress small scale power and show how they scale as a function of wave number, k.

(d) Explain the origin of CMB polarization. How large is it compared to the CMB anisotropy? Explain why the peaks in the polarization power spectrum are shifted with respect to the anisotropy peaks.

(e) Microwave background fluctuations can also be created in the low redshift universe by non-linear effects. Give three such non-linear contributions and describe the physical origin and frequency spectrum of each.

(f) Describe two sources, galactic or extra-galactic, of foregrounds for the CMB. For which frequencies does each dominate over the CMB anisotropies?

2 The Boltzmann equation is most naturally described in terms of the conformal position, x^i, and its canonical momentum, $\tilde{p}_i = mg_{i\nu}dx^\nu/ds$. In these variables, the number of particles in a given element of phase space is given by $dN = f(t, \mathbf{x}, \tilde{\mathbf{p}})\, d^3\mathbf{x} d^3\tilde{\mathbf{p}}$.

It is useful to define an alternative momentum, p_i, which obeys the standard Minkowski space relation between momentum and energy, $p_0^2 = p^2 + m^2$. This has comoving momentum, q, and direction vectors, n_i, given by $p_i = p^i \equiv aqn_i$, where a is the scale factor. In these variables, the collisionless Boltzmann equation is given by

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial q}\frac{dq}{dt} + \frac{\partial f}{\partial n^i}\frac{dn^i}{dt} = 0.$$

(a) For small perturbations, the distribution function can be decomposed into a uniform piece and a first order perturbation: $f = f_0 + f_1$. What is the leading order ($0^{th}, 1^{st}, 2^{nd}, ...$) of each of factors in the above equation. Use these considerations to write the first order Boltzmann equation for f_1.

(b) In Fourier space, the scalar piece of the metric perturbation can be written as

$$h_{ij} = \tfrac{1}{3}\delta_{ij}h + (\hat{k}_i\hat{k}_j - \tfrac{1}{3}\delta_{ij})h_S.$$

Using this decomposition and the geodesic equations,

$$\tilde{p}_0\frac{d\tilde{p}_\nu}{dt} = \frac{1}{2}g_{\alpha\beta,\nu}\tilde{p}^\alpha\tilde{p}^\beta,$$

rewrite the collisionless Boltmann equation for massless photons to show

$$\Delta' + ik\mu\Delta = \frac{2}{3}(h' - h_S') + 2\mu^2 h_S'.$$

Here, $\Delta(k, \mu, \tau) \equiv -4f_1\left(q\frac{\partial f_0}{\partial q}\right)^{-1}$, $\mu \equiv \hat{k}\cdot\hat{n}$ and primes denote derivatives with respect to comoving time, $d\tau = dt/a$.

(c) In the matter dominated regime, $h = h_S + C = \frac{1}{4}Ak^{1/2}\tau^2$, where A and C are constants. If the distribution function immediately after last scattering was given by $\Delta(\tau_{LS})$, solve for the distribution function today. Which term dominates the large angle anisotropies?

3 (a) Sketch a Penrose-Carter diagram for maximally extended anti de Sitter space and label all the infinities present in the diagram.

(b) Define what it means for a spacetime to be globally hyperbolic. (A qualitatative, physical definition suffices here).

(c) Is maximally extended anti de Sitter space globally hyperbolic? Why or why not?

(d) Copy the diagram from part (a) and place an arbitrary point P in AdS inside the diagram. Sketch the family of all timelike geodesics passing through P.

(e) Copy the diagram from part (a) again and indicate on your conformal diagram the region covered by the Randall-Sundrum coordinates endowed with the line element

$$ds^2 = dy^2 + \exp(-2y)\left[-dt^2 + dx_1{}^2 + dx_2{}^2 + dx_3{}^2\right]. \qquad (*)$$

[Assume that the x_1, x_2, and x_3 directions are perpendicular to the page.]

(f) Compute the trajectory of the most general timelike geodesic using the metric $(*)$. Parameterize the trajectory in terms of the proper time. Compute the acceleration of the surface defined by $y = 0$.

4 Consider the two following scenarios for a universe with large extra dimensions.

(i) Suppose that we live on a (3+1)-dimensional braneworld of the kind in the infinite Randall-Sundrum scenario—that is, there is a large extra "fifth" dimension y for which $y > 0$, with our brane situated at $y = 0$, a Z_2 orbifold symmetry about our brane, and the line element

$$ds^2 = dy^2 + \exp(-2y)\left[-dt^2 + dx_1{}^2 + dx_2{}^2 + dx_3{}^2\right]$$

for the five-dimensional bulk that surrounds us.

(ii) Suppose that we live in a spacetime with N extra spatial dimensions with the topology $(S^1)^N$ and characteristic size R, so that at low energies all the fields of the standard model are confined to a (3+1)-dimensional brane but gravity explores the full $(N+3)+1$ dimensional spacetime.

Please write a brief essay listing and explaining as many different ways as possible for detecting observationally or experimentally the presence of extra dimensions for both scenarios. Your answer does not require any detailed mathematical derivations, nor specific formulae. However, you should explain as clearly as possible the physics underlying each of these tests in a qualitative manner. For the first scenario consider the generalization to an expanding universe and its observational consequences as well.