UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 7 June 2001 1.30 to 3.30

PAPER 56

COMPUTER-AIDED GEOMETRIC DESIGN

*Answer any **FOUR** questions. The questions carry equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 If points in a projective space are represented by their homogeneous coordinates, lines may be represented by the Plucker coordinates expressed in an antisymmetric matrix.

How may the matrix L^{ij} be computed from two points through which the line passes ?

How are the following related to the line representations L_{ij} and L^{ij} :–

(a) the points in which the line cuts the coordinate planes ?

(b) the planes containing the line and the coordinate axis directions ?

How are the two forms L_{ij} and L^{ij} related to each other ?

How may the point P of intersection of the line L^{ij} with a general plane F_i be computed ?

2 What are the important classes of transformations within the class of general linear (projective) transformations ?

What invariants characterise each of them ?

Which of the following surface forms remain of the same form under each of them:– sphere, circular cylinder, general cone, paraboloid ?

3 The fundamental definition of a B-spline basis function of degree d over a given knot vector t_i is a piecewise polynomial of maximal continuity $(d-1)$ and minimal support ($d+1$ spans). A normalised B-spline basis function has an integral equal to the length of its support interval divided by the number of spans it covers, and the sum of the normalised basis functions is unity everywhere.

Using these definitions, express the derivative of a B-spline basis function of degree d in terms of those of degree $d-1$.

Using this result, show that if each of the control points of an unequal interval parametric B-spline curve has an x-coordinate equal to the average of the d consecutive interior knots of the corresponding basis function, the x-coordinate of every point of the curve will be equal to its parameter value.

You can assume that the knot vector has a multiplicity of $d+1$ at each end (Bézier end conditions).

4 What are the basic enquiries which a parametric surface definition needs to be able to support ?

How do you determine the surface normal at a point on a parametric surface ?

Using these enquiries, write down an algorithm for computing the curve of intersection of two parametric surfaces $S_1(u_1, v_1)$ and $S_2(u_2, v_2)$.

5 What are the basic enquiries which a recursive subdivision surface definition needs to be able to support ?

Using these enquiries, write down an algorithm for computing the curve of intersection of two recursive subdivision surfaces S_1 and S_2.

6 A piece of surface consists of a collection of triangles all meeting at a single vertex P, and meeting in pairs at the edges incident on P. The sides of the triangles opposite this vertex thus form a single polygonal loop of boundary, joining the other vertices B_i in sequence.

Express the algebraic area of the surface projected in the direction of the unit vector V (if V penetrates a triangle from one side the contribution to the area is positive, from the other negative) in terms of V, P and B_i, and thus show that it depends only on the boundary polygon, not on the position of the common vertex P.

What direction V results in the largest algebraic area, and what directions give zero algebraic area ?