UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 30 May 2003 9 to 11

PAPER 10

CLASSICAL BANACH SPACES

Attempt **THREE** *questions.*

There are **five** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Suppose that X is an infinite-dimensional subspace of l_p (where $1 \leqslant p < \infty$). Show that X contains an infinite-dimensional subspace Y which is complemented in l_p and isomorphic to l_p.

Suppose that Z is a complemented infinite-dimensional subspace of l_p (where $1 \leqslant p < \infty$). Show that Z is isomorphic to l_p.

[You may use any properties of normalised block basic sequences that you may need.]

2 Suppose that $x_1, \ldots, x_n$ are vectors in a Banach space X and that $\epsilon_1, \ldots, \epsilon_n$ are Bernoulli random variables. Show that

$$\left\| \sum_{j=1}^{n} \epsilon_j x_j \right\|_{L^2(X)} \leqslant \sqrt{2} \left\| \sum_{j=1}^{n} \epsilon_j x_j \right\|_{L^1(X)} .$$

What does it mean to say that a Banach space has *cotype 2*? Show that $L^1(0,1)$ has cotype 2.

3 State and prove Pietsch's factorization theorem concerning p-summing operators.

Show that if $f \in L^p(0,1)$ then the multiplication operator $M_f : C([0,1]) \to L^p(0,1)$ defined by $M_f(g) = fg$ is p-summing.

4 State and prove Grothendieck's inequality.

Show that every bounded linear mapping from $L^1(0,1)$ to a Hilbert space is absolutely summing.

Show that the closed linear span of the Rademacher functions in $L^1(0,1)$ is not complemented in $L^1(0,1)$.

5 Suppose that a Banach space Y has cotype 2. Show that there are constants K and L such that if $T \in L(l_\infty^N, Y)$ then $\pi_2(T) \leqslant K\pi_4(T)$ and $\pi_2(T) \leqslant L\,\|T\|$.

Suppose that E is an N-dimensional subspace of Y. Show that the Banach-Mazur distance $d(E, l_\infty^N)$ satisfies $d(E, l_\infty^N) \geqslant L^{-1}\sqrt{N}$.

[You may use any result about the 2-summing norm of the identity mapping of a finite-dimensional normed space that you may need.]