UNIVERSITY OF CAMBRIDGE

macs

MATHEMATICAL TRIPOS Part III

Thursday 9 June, 2005 9 to 11

PAPER 52

BRANES

*Answer **ALL** questions in Section A.*

*Answer any **ONE** of the two questions in Section B.*

Section A carries 2/3 credit.

Section B carries 1/3 credit.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

Section A

1 Show that the standard supersymmetry anticommutator

$$\{Q_\alpha, Q_\beta\} = (C\Gamma_m)^m P_m$$

would imply, if valid, that only configurations with $P^2 = 0$ can be supersymmetric. How is this conclusion evaded in supersymmetric field theories that admit 1/2 supersymmetric brane solutions? Why might your result lead you to suspect the existence of 1/2 supersymmetric membrane and fivebrane solutions of 11-dimensional supergravity?

2 Given that a supersymmetric brane action is invariant under the fermionic gauge invariance called "kappa-symmetry", explain why the condition for a given brane configuration to preserve some fraction of supersymmetry is

$$\Gamma\epsilon = \epsilon,$$

and explain how this equation determines the fraction of supersymmetry preserved [You will need the form of the kappa symmetry transformation in terms of Γ but you are not expected to derive it]. What is the matrix Γ for the 11-dimensional supermembrane.

3 Write down the bosonic truncation of the action for a supermembrane coupled to the metric and 3-form potential A of 11-dimensional supergravity.

The M2-brane solution of 11-dimensional supergravity for metric and 4-form field strength F is

$$ds^2_{11} = H^{-\frac{2}{3}} ds^2\left(E^{(1,2)}\right) + H^{\frac{1}{3}} ds^2\left(E^8\right), \qquad F = \mathrm{vol}(E^{(1,2)}) \wedge dH^{-1},$$

where H is a harmonic function on E^8 with isolated singularities. Why does this imply the absence of a force between parallel membranes? Confirm this "no-force" condition by considering the action for a probe membrane in the M2 background.

4 The array

$$\begin{array}{ll} M2: & 1\ 2 \\ M5: & 1\ \ \ 3\ 4\ 5\ 6 \end{array}$$

represents an M2-brane in the 12 plane ending on an M5-brane in the 13456 5-plane. Why would you expect the minimum energy configuration of this type to preserve 1/4 supersymmetry. Use string/M dualities (any properties of which you may state without proof) to deduce the existence of a 1/4 supersymmetric IIB superstring configuration in which a D-string ends on a D3-brane.

Section B

5 Write an essay with the title *Effective actions for field theory branes.* Your essay should begin with an explanation of why Goldstone fields dominate the low energy dynamics and why, in the case of a bosonic field theory, these fields are governed by Dirac's brane action [You are not expected to derive this action; instead you should motivate it by symmetry and dimensional analysis]. You should verify, via the Monge gauge, that the linearized fluctuations are transverse waves. You should then explain why a modification of the Dirac action is needed for branes in supersymmetric field theories, and why the effective action must include a WZ term [You should concentrate on the properties of this term, and its implications, *not* on the details of its form or how it is constructed].

6 Write an essay with the title *String theory branes from M-branes.* Your essay should give a general overview of how branes in M-theory imply the existence of branes in IIA string theory, and how these are related via T-duality to branes in IIB string theory. You should illustrate your overview by giving some details of how the IIA string and D2-brane follow from the M2-brane, ignoring fermions but taking care to deduce the dependence on the IIA string coupling constant g_s. [You will need the following result for an 11-dimensional metric with $U(1)$ Killing vector field $\partial/\partial Y$ in terms of string-frame 10-dimensional fields of IIA supergravity:

$$ds_{11}^2 = e^{-\frac{2}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (dY - C)^2].$$

END OF PAPER