

PAPER 14

BASIC ALGEBRAIC GEOMETRY

*Answer any **THREE** questions.*

Here k denotes an algebraically closed field of characteristic $\neq 2, 3$.

You may not start to read the questions
printed on the subsequent pages until
instructed to do so by the Invigilator.

- 1** (a) Let $C, D \subset \mathbb{P}_k^2$ be curves of degrees n, m . Show that $|C \cap D| \leq n \cdot m$ unless C, D have a common component.

[You may use standard properties of the resultant without proof.]

- (b) Let $C \subset \mathbb{P}_k^2$ be a reduced and irreducible curve of degree d with singular points $P_i \in C$ of multiplicities m_i . Show that $\sum m_i(m_i - 1) \leq (d - 1)(d - 2)$.
- (c) Let $C \subset \mathbb{P}_k^2$ be a reduced and irreducible quartic with three double points. Show that C is rational.

- 2** Let $C \subset \mathbb{P}_k^2$ be a nonsingular plane cubic. Show that

- (a) C has an inflection point.
 (b) C is described, in suitable coordinates

$$y^2 = x(x - 1)(x - \lambda).$$

- (c) C is not rational.

- 3** Let $S \subset \mathbb{P}_k^3$ be a nonsingular cubic surface. Assuming that S contains a line, show that:

- (a) There are at most three lines through any point of S .
 (b) Given a line $l \subset S$, there are five pairs $\{l_i, l'_i\}$ of lines on S such that:
 (i) $l \cup l_i \cup l'_i$ is contained in a plane, for all i ;
 (ii) if $i \neq j$, $\{l_i \cup l'_i\} \cap \{l_j \cup l'_j\} = \emptyset$.
 (c) S is birational to \mathbb{P}_k^2 .

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Write an essay on dimension theory. Make sure you state the theorem on the dimension of fibres of a morphism, and you include the outline of some applications, such as the existence of a line on a nonsingular cubic surface.