

MATHEMATICAL TRIPOS      Part III

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Friday 8 June 2007    1.30 to 4.30

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PAPER 23

ALGEBRAIC CURVES

Attempt **THREE** questions.

There are **SIX** questions in total.

The questions carry equal weight.

*In answering any question, you may assume any result that  
you would have proved in answering a previous question.*

*Throughout,  $k$  is a field.*

**STATIONERY REQUIREMENTS**

*Cover sheet  
Treasury Tag  
Script paper*

**SPECIAL REQUIREMENTS**

*None*

<p><b>You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.</b></p>
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- 1** Describe  $\mathbb{P}_{\mathbb{Z}}^n$  in terms of a functor, via quotients of a free module of rank  $n + 1$ .  
Use your answer to derive the Euler sequence for the tangent sheaf of  $\mathbb{P}^n$  and compute  $\Omega_{\mathbb{P}^1}^1$ .
  
- 2** Describe the computation of  $H^i(\mathbb{P}_k^1, \mathcal{O}(n))$  in terms of a Čech complex, and deduce a duality theorem for the cohomology of locally free coherent sheaves on  $\mathbb{P}_k^1$ .
  
- 3** Prove that any locally free coherent sheaf  $\mathcal{E}$  on  $\mathbb{P}_k^1$  is of the form  $\mathcal{E} \cong \mathcal{O}(a_1) \oplus \dots \oplus \mathcal{O}(a_n)$ .
  
- 4** Show that any smooth projective curve  $C$  over  $k$  has a dualizing sheaf  $D_C$  (for locally free sheaves) and that  $D_C \cong \Omega_C^1$ . [*Any algebraic identities you need may be quoted without proof.*]
  
- 5** Suppose that  $C$  is a smooth projective curve. Show that  $H^i(C, \mathcal{E}) = 0 \forall i \geq 2$  and for all locally free sheaves  $\mathcal{E}$  on  $C$ .  
Define the genus of  $C$ , and state and prove the Riemann–Roch theorem for curves.
  
- 6** Assume  $k = \bar{k}$ . Define the functors  $\underline{Pic}_C^d$ . Show how to represent them. Any results you need about the semi-continuity of cohomology groups may be assumed.

**END OF PAPER**