UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 12 June 2001 9 to 12

PAPER 4

ADVANCED TOPICS IN COMMUTATIVE ALGEBRA

*Answer **ALL** questions. They are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Describe the construction of Cech cohomology groups of a coherent sheaf on a projective scheme in terms of a given affine cover and compute the cohomology groups $H^i(P^r_k, O(n))$, where k is a field.

[*You need only sketch the key steps in the calculation.*]

2 Construct an exact sequence

$$0 \to \Omega^1 \to O(-1)^{n+1} \to O \to 0$$

of sheaves on projective space P^n and compute the determinant $\wedge^n \Omega^1$.

3 Take a conic C defined over an algebraically closed field k. Embed C in a plane L and embed L as a hyperplane in P^3_k. Find numbers a, b such that, when C is identified with P^1, the normal sheaf N_{C/P^3} is $O(a) \oplus O(b)$.

4 Suppose that X is a projective scheme over a field k. Describe the Hilbert functor associated to X. Now suppose that Z is a closed subscheme of X, also defined over k. Explain what is the tangent space to the Hilbert scheme of X at the point corresponding to Z in terms of the scheme *Spec* $k[t]/(t^2)$ and then describe this tangent space in terms of the normal sheaf of Z.

[*You may take for granted the existence of the Hilbert scheme.*]