

# Probability & Statistics 1 2013–14

## Progress Test

24 January 2014

**Answer all 5 questions.**

**The total number of marks available is 100.**

### QUESTION 1 [24 marks]

An insurance company knows that the size of a flood damage claim (measured in units of £1000) is a random variable,  $X$ , with density function  $f(x)$  equal to 0 for  $x < 0.5$  and otherwise of the form

$$f(x) = \frac{a}{(x+2)^3}, \quad x > 0.5.$$

- (i) Calculate the value of the constant  $a$ . [4 marks]
- (ii) Evaluate the distribution function of  $X$  and hence calculate the median of  $X$  (i.e., the value  $m$  such that  $\mathbb{P}(X \leq m) = 0.5$ .) [6 marks]
- (iii) Evaluate the expectation of  $X$ ; comment on the skewness of this distribution. [6 marks]
- (iv) When a claim is for more than £10,000 a reinsurance company pays part of the claim. If 10 claims are received in one day, what is the probability that the reinsurance company will pay out on more than 1 of them? [8 marks]

### QUESTION 2 [16 marks]

A linguist conducts a survey of 120 languages on the island of New Guinea.

- (i) Each language, independently, has probability  $\frac{2}{3}$  of making use of the sound ‘z’. What is the probability that the linguist has to survey between 3 and 5 languages (inclusive) before finding one which uses the ‘z’ sound? [6 marks]
- (ii) The sound ‘th’ is rarely encountered: only 3% of New Guinea languages use it.
  - (a) By using a suitable approximation, or otherwise, find the probability that 5 or more languages in the survey use the ‘th’ sound. [5 marks]
  - (b) The 120 languages are divided into 12 geographical groups of 10 languages each. Find the expected number of language groups which contain at least one language which uses ‘th’. [5 marks]

### QUESTION 3 [14 marks]

22% of voters support the Conservatives, 30% Labour, 14% some other party, with 34% not supporting a political party. When asked what party they support, voters either answer truthfully or refuse to answer; the probability of a refusal is 25% for a Conservative supporter, 20% for a Labour supporter, or 5% for everyone else.

- (i) What proportion of voters say that they support the Conservatives? [4 marks]
- (ii) Of those who do answer, what proportion claim to be Conservative supporters? [5 marks]
- (iii) Of those who refuse to answer, what proportion actually support a political party? [5 marks]

**QUESTION 4** [26 marks]

A bag contains an unknown number of balls, of which some are red, some are green and the rest are blue. Two balls are taken out of the bag (without replacement). Events  $A$ ,  $B$  and  $C$  are defined as follows:

- $A$  = “At least one of the balls is blue”;
- $B$  = “Neither of the balls is red”;
- $C$  = “More green balls than blue balls are obtained”.

- (i) List all the possible outcomes of the experiment, stating which belong to  $A$ , which to  $B$  and which to  $C$ . [6 marks]
- (ii) Draw a Venn diagram to illustrate the relationship between  $A$ ,  $B$  and  $C$ , marking in on the diagram all the possible outcomes. [6 marks]
- (iii) Suppose the bag contains 10 balls, of which one is blue,  $r$  red and  $9 - r$  green.
  - (a) Calculate  $\mathbb{P}(A)$ ,  $\mathbb{P}(B)$ ,  $\mathbb{P}(C)$  and  $\mathbb{P}(A \cup B \cup C)$ .
  - (b) Find the value of  $r$  which makes  $B$  and  $C$  independent of each other. [14 marks]

**QUESTION 5** [20 marks]

An archaeological excavation has turned up the remains of 25 mammoths, which can be divided into three types — A, B and C — according to their bone structure. From the remains recovered the heights of the 25 mammoths can be reliably determined, as shown in the table below:

	Heights of mammoths (cm)												
Type A	411	425	372	422	406	295	398	408	361	424	415	384	
Type B	348	381	362	299	361	374	331						
Type C	379	391	404	305	322	377							

- (i) Calculate the median and quartiles of each of the three samples. [6 marks]
- (ii) Identify any outliers in each of the three samples. [4 marks]
- (iii) Construct a box-and-whisker plot to provide a visual comparison of the heights of the three types of mammoth. [6 marks]
- (iv) Comment on the plot in terms of location and spread. [4 marks]

Formulae

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C).$$
$$\mathbb{P}(A | B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B | A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Distribution	Notation	Mean	Variance	$p(x)$	Domain
Binomial	$\text{Bin}(k, \theta)$	$k\theta$	$k\theta(1 - \theta)$	$\binom{k}{x} \theta^x (1 - \theta)^{k-x}$	$x = 0, 1, \dots, k$
Poisson	$\text{Pois}(\lambda)$	$\lambda$	$\lambda$	$e^{-\lambda} \frac{\lambda^x}{x!}$	$x = 0, 1, \dots$
Geometric	$\text{Geom}(\theta)$	$\theta^{-1}$	$\theta^{-2}(1 - \theta)$	$\theta(1 - \theta)^{x-1}$	$x = 1, 2, \dots$