Probability and Statistics 1 - Surgery Hours class (Andres Villegas)
Exercise Sheet 8: Continuous Random Variables 2

1. On May 5, in a certain city, temperatures have been found to be normally distributed with mean $\mu = 24°C$ and variance $\sigma^2 = 9$. The record temperature on that day is $27°C$.
   a) What is the probability that the record of $27°C$ will be broken next May 5?
   b) What is the probability that the record of $27°C$ will be broken at least 3 times during the next 5 years on May 5? (Assume that the temperatures during the next 5 years on May 5 are independent.)
   c) How high must the temperature be to place it among the top 5% of all temperatures recorded on May 5?

2. Let Z be the standard normal random variable. If $z > 0$ and $F_Z(z) = \alpha$, what are $F_Z(-z)$ and $P(-z \leq Z \leq z)$?

3. A machine used to automatically fill 355ml water bottles. The actual amount put into each bottle is a normal random variable with mean 360ml and standard deviation of 4ml.
   a) What proportion of bottles are filled with less than 355ml of water?
   b) Suppose that the mean fill can be adjusted. To what value should it be set so that only 2.5% of bottles are filled with less than 355ml?

4. Suppose that a local vote is being held to see if a new manufacturing facility will be built in the locality. A polling company will survey 200 individuals to measure support for the new facility. If in fact 53% of the population oppose the building of this facility, use the normal approximation to the binomial, with a continuity correction, to approximate the probability that the poll will show a majority in favour?

5. Customers arrive randomly and independently at a service window, and the time between arrivals has an exponential distribution with a mean of 12 minutes. Let X equal the number of arrivals per hour. What is $P(X = 10)$? (Hint: When the time between successive arrivals has an exponential distribution with mean $\frac{1}{\lambda}$ (units of time), then the number of arrival per unit time has a Poisson distribution with parameter (mean) λ).

6. An investment account earns an annual interest rate R that follows a uniform distribution on the interval $(0.04, 0.08)$. The value of a 10,000 initial investment in this account after one year is given by $V = 10{,}000e^R$. Determine the cumulative distribution function, $F_V(v)$ of V.

7. An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$, where X is an exponential random variable with mean 1 year. Determine the probability density function $f_Y(y)$, for $y > 0$, of the random variable Y.