Probability and Statistics 1 - Surgery Hours class (Andres Villegas)
Exercise Sheet 7: Continuous Random Variables 1

1. Let X_1, X_2, X_3 be three independent, identically distributed random variables each with density function

$$f(x) = \begin{cases} ax^2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

   a) Determine the value of a.
   b) Let $Y = \max\{X_1, X_2, X_3\}$. Find $P\left(Y > \frac{1}{2}\right)$.

2. Petrol is delivered to a garage every Monday morning. The weekly demand for petrol at this garage, in thousands of gallons, is a continuous random variable X distributed with a probability density function of the form

$$f(x) = \begin{cases} kx(c - 2x) & \frac{1}{2} \le x \le \frac{3}{2} \\ 0 & otherwise \end{cases}$$

   a) Given that the mean weekly demand is 900 gallons, determine the values of k and c.
   b) Calculate the mean number of gallons **sold** per week at this garage if its supply tanks are filled to their total capacity of 1000 gallons every Monday morning.

3. Let T be the time from birth until death of a randomly selected member of a population. Assume that T has a uniform distribution on $[0,100]$ (This model for mortality is known in the actuarial context as the DeMoivre's Mortality law).
   a) Find the life expectancy of the population $E(T)$ and the corresponding variance $V(T)$
   b) What is the probability that a randomly selected member of the population survives beyond age 57.
   c) Suppose that you consider the subset of the population who survive to age 40. Let S be the random variable for the age at time of death of these survivors. Show that S has a uniform distribution over $[40,100]$ (Hint: Show that $P(S > t) = P(T > t | T > 40) = \frac{100-t}{60}$).
   d) Find $E(S)$ and $P(S > 57)$ (Compare this with the results in a) and b)).

4. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

5. You are given the following information about N, the annual number of claims for a randomly selected insured:

$P(N = 0) = \frac{1}{2}$	$P(N = 1) = \frac{1}{3}$	$P(N > 1) = \frac{1}{6}$

   Let S denote the total annual claim amount for an insured. When $N = 1$, S is exponentially distributed with mean 5. When $N > 1$ S is exponentially distributed with mean 8.
   Determine $P(4 < S < 8)$