Probability and Statistics 1 - Surgery Hours class (Andres Villegas)
Exercise Sheet 5: Discrete Random Variables 2

1. A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay 1000 for each day, up to 2 days, that the opening game is postponed. The insurance company determines that the number of consecutive days of rain beginning on April 1 is a Poisson random variable with mean 0.6. What is the mean of the amount the insurance company will have to pay? What is the standard deviation of this amount?

2. A company buys a policy to insure its revenue in the event of major snow-storms that shut down business. The policy pays nothing for the first such snowstorm of the year and £10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

3. At a medical clinic, patients are given X-rays to test for tuberculosis.
   a) If 15% of these patients have the disease, what is the probability that on a given day the first patient to have the disease will be the fifth one tested?
   b) What is the probability that the first with the disease will be the tenth one tested?
   c) What is the mean number of patients that need to be test to find the first patient with tuberculosis?
   d) What is the probability that the fifteenth patient tested will be the third with tuberculosis?
   e) What is the mean number of patients without tuberculosis tested before the sixth patient with tuberculosis is tested?

4. A telemarketer makes successful calls with probability 0.20.
   a) What is the probability that her fifth sale will be on her sixteenth call?
   b) If each sale made by the telemarketer is for £250, what is the mean number of total calls she will have to make to reach £2,000 in total sales?

5. A large portfolio of policies is such that a proportion p $(0 < p < 1)$ incurred claims during the last calendar year. An investigator examines a randomly select group of 25 policies from the portfolio.
   a) Use a Poisson approximation to the binomial distribution to calculate an approximate value for the probability that there are at most 4 policies with claims in the two cases where i) $p = 0.1$ and ii) $p = 0.2$.
   b) Comment briefly on the above approximations, given that the exact values of the probabilities in part a), using the binomial distribution, are 0.9020 and 0.4207 respectively.