

Probability and Statistics 1 - Surgery Hours class (Andres Villegas)

Exercise Sheet 5: Discrete Random Variables 1

1. A contestant on a game show selects a ball from an urn containing 25 balls numbered from 1 to 25. His prize is £1,000 times the number of the ball selected. If X is the random variable for the amount he wins, find the mean and standard deviation of X .
2. A simple procedure for incorporating a “no claims discount” into an annual insurance policy is as follows:
 - a premium of £400 is payable for the first year;
 - if no claims are made in the first year, the premium for the second year is £400 k , where k is a constant such that $0 < k < 1$;
 - if no claim is made in the first year, the premium for the third year is £400 k^2 ;
 - if no subsequent claims are made in future years, the premium remains as £400 k^2 ;
 - if a claim is made, the premium the following year reverts to £400 and the procedure starts again as above.
 - a) Show that the probability distribution of the premium for the fourth year, that is, for the year following the third year, is given by
 - £400 with probability p
 - £400 k with probability $p(1 - p)$
 - £400 k^2 with probability $(1 - p)^2$where p is the probability of a claim being made in any year.
 - b) Obtain an expression for the expected premium for the fourth year under this procedure.
 - c) If it is desired that this expected premium should equal £300, determine the required value of k for the case where $p = 0.1$.
3. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist.
 - a) What is the expected revenue of the tour operator?
 - b) Is overbooking a profitable strategy for the tour operator?
 - c) Find the minimum probability of no-show p that would make overbooking a profitable strategy.
4. A study is being conducted in which the health of two independent groups of ten people is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants).

What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?
5. A hospital receives $\frac{1}{5}$ of its flu vaccine shipments from a Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective. What is the probability that this shipment came from Company X?
6. A company establishes a fund of 120 from which it wants to pay an amount, C , to any of its 20 employees who achieve a high performance level during the coming year. Each employee has 2% chance of achieving a high performance level during the coming year, independent of any other employee. Determine the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.