Probability and Statistics 1 - Surgery Hours class (Andres Villegas)
Exercise Sheet 4: Conditional probability, Bayes' Theorem, Permutation and Combinations

1. Let $X,\ Y, Z$ be three coins in a box. Suppose X is a fair coin, Y is two-headed, and Z is weighted so that the probability of heads is $\frac{1}{3}$. A coin is selected at random is tossed.
   a) Find the probability that heads appears, that is, find $P(H)$.
   b) If heads appears, find the probability that is the fair coin X, that is, find $P(X|H)$.
   c) If tails appears, find the probability it is the coin Z, that is, find $P(Z|T)$.

2. There are 10 people, consisting of 5 couples.
   a) How many ways are there to seat them in a row?
   b) How many ways are there to seat them at a circular dinner table?
   c) How many ways are there to seat them in a row if all couples are to get adjacent seats?
   d) How many ways are there to seat them at a circular dinner table if all couples are to get adjacent seats?

3. In a room containing n people, calculate the chance that at least two of them have the same birthday. Calculate this probability for $n = 40$. Is it bigger or smaller than you expected?

4. A quiz has 5 multiple-choice questions. Each question has 4 answer choices, of which 1 is correct and the other 3 are incorrect. Suppose that you guess on each question.
   a) How many ways are there to answer the 5 questions?
   b) What is the probability of getting all 5 question right?
   c) What is the probability of getting exactly 4 questions right and 1 wrong?
   d) What is the probability of doing well (getting at least 4 right)?

5. An insurance company divides its policyholders into low-risk and high-risk classes. For the year, of those in the low-risk class, 80% had no claims, 15% had one claim, and 5% had 2 claims. Of those in the high-risk class, 50% had no claims, 30% had one claim, and 20% had two claims. Of the policyholders, 60% were in the low-risk class and 40% in the high-risk class.
   a) If a policyholder had no claims in the year, what is the probability that he is in the low-risk class?
   b) If a policyholder had two claims in the year, what is the probability that he is in the high-risk class?

6. A test for Alzheimer's disease is 95 percent effective in detecting the disease when it is present, but also gives positive results 10 percent of the time when it is not present (false positive). Suppose 4 percent of the population over 65 years have Alzheimer's disease.
   a) What is the probability that a person over 65 years chosen at random will test positively for the disease?
   b) Suppose a person over 65 tests positively. What is the probability that the person has the disease?
   c) Suppose a person over 65 tests negatively. What is the probability that the person has the disease?