## Probability and Statistics 1 - Surgery Hours class (Andres Villegas) Exercise Sheet 3: Axioms of probability, independence and conditional probability

1. James is bidding online for a bicycle and a computer. He believes there is $60 \%$ chance of getting the bicycle (event $B$ ) and a $40 \%$ chance of getting the computer (event $C$ ).
a) Calculate the probability that neither offer will be accepted if $B$ and $C$ are independent events.
b) Calculate the probability that neither offer will be accepted if $B$ and $C$ are mutually exclusive events.
c) What is the largest possible probability that neither offer will be accepted?
2. Using the axioms of probability prove that for any two event $A$ and $B$
a) $P(A \backslash B)=P(A)-P(A \cap B)$ (Recall that $A \backslash B=A \cap B^{c}$ )
b) $\quad P(A \cup B)=P(A)+P(B)-P(A \cap B)$
(Hint: Use the fact that $A \cup B=(A \backslash B) \cup(A \cap B) \cup(B \backslash A)$ )
3. Let $A$ and $B$ be independent events. Show that
a) $A$ is independent of $B^{c}$.
b) $A^{c}$ is independent of $B^{c}$.
4. A dice is rolled repeatedly until a 6 turns up. Show that event $A$ that "a 6 will eventually show up", is certain to occur.
5. An insurance company classifies its claims as low if they are under $£ 10,000$, and high otherwise. During the year $79.2 \%$ of its policyholders filed no claims, $16.9 \%$ filed low claims, and $3.9 \%$ filed high claims. If a policyholder filed a claim, what is the probability that it was a low claim?
6. In modelling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p_{n+1}=\frac{1}{5} p_{n}$, where $p_{n}$ represents the probability that the policyholder files $n$ claims during the period. Under this assumption, what is the probability that a policyholder files more than one claim during the period?
7. The question, "Do you smoke" was asked to 100 people. Results are shown in the table

|  | Yes | No | Total |
| :--- | :--- | :--- | :--- |
| Male | 19 | 41 | 60 |
| Female | 12 | 28 | 40 |
| Total | 31 | 69 | 100 |

a) What is the probability of a randomly selected individual being a male who smokes?
b) What is the probability of a randomly selected individual being a male?
c) What is the probability of a randomly selected individual smoking?
d) What is the probability of a randomly selected male smoking?
e) What is the probability that a randomly selected smoker is male?

