Probability and Statistics 1 - Surgery Hours class (Andres Villegas) Exercise Sheet 3: Axioms of probability, independence and conditional probability

- 1. James is bidding online for a bicycle and a computer. He believes there is 60% chance of getting the bicycle (event B) and a 40% chance of getting the computer (event C).
 - a) Calculate the probability that neither offer will be accepted if B and C are independent events.
 - b) Calculate the probability that neither offer will be accepted if *B* and *C* are mutually exclusive events.
 - c) What is the largest possible probability that neither offer will be accepted?
- 2. Using the axioms of probability prove that for any two event *A* and *B*
 - a) $P(A \setminus B) = P(A) P(A \cap B)$ (Recall that $A \setminus B = A \cap B^c$)
 - b) $P(A \cup B) = P(A) + P(B) P(A \cap B)$ (Hint: Use the fact that $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$)
- 3. Let *A* and *B* be independent events. Show that
 - a) A is independent of B^c .
 - b) A^c is independent of B^c .
- 4. A dice is rolled repeatedly until a 6 turns up. Show that event *A* that "a 6 will eventually show up", is certain to occur.
- 5. An insurance company classifies its claims as low if they are under £10,000, and high otherwise. During the year 79.2% of its policyholders filed no claims, 16.9% filed low claims, and 3.9% filed high claims. If a policyholder filed a claim, what is the probability that it was a low claim?
- 6. In modelling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \ge 0$, $p_{n+1} = \frac{1}{5}p_n$, where p_n represents the probability that the policyholder files n claims during the period. Under this assumption, what is the probability that a policyholder files more than one claim during the period?
- 7. The question, "Do you smoke" was asked to 100 people. Results are shown in the table

	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

- a) What is the probability of a randomly selected individual being a male who smokes?
- b) What is the probability of a randomly selected individual being a male?
- c) What is the probability of a randomly selected individual smoking?
- d) What is the probability of a randomly selected male smoking?
- e) What is the probability that a randomly selected smoker is male?