

Mathematical Sciences

BACHELOR DEGREE EXAMINATIONS

MAY 2006

MA1972

DISCRETE MATHEMATICS, PROBABILITY AND STATISTICS

Time allowed: TWO hours plus five minutes reading time.

Answer **ALL** questions from **SECTION A**.

Answer **TWO** questions from **SECTION B**. If more than **TWO** questions from **Section B** are answered, marks from the best **TWO** answers will be counted.

Section A carries 50% of the total marks available for the paper.

All questions in **Section B** carry equal marks.

An indication of the marks allocated to each sub-section of a question is shown in brackets in the right hand margin.

Additional Information:

Neave statistical tables will be provided.

Section A

- A1 A computer password consists of three digits followed by two letters. How many different passwords are possible? [4 marks]
- A2 An urn contains four red balls and two blue balls. A ball is selected at random from the urn. Without replacing the first ball, a second ball is then selected at random from the urn.
- a. Find the probability that the first ball is red. [3 marks]
 - b. Find the probability that the second ball is red. [2 marks]
 - c. Find the probability that both the first and second balls are red. [4 marks]
- A3 Let A, B, C be sets with $A \subseteq B$.
- a. What is $A \cap B$? [2 marks]
 - b. Prove that $A \cap C \subseteq B \cap C$. [3 marks]
- A4 a. Find $\mathcal{P}(\{0, 1\})$ and $\mathcal{P}(\{\emptyset, 1\})$. [4 marks]
- b. Let $A = \{1, 2, \dots, m\}$ and $B = \{1, 2, \dots, n\}$. Find $|\mathcal{P}(A \times B)|$. [3 marks]
- A5 A random sample of 16 measurements is to be drawn from a population that is Normal with a mean of 4 and standard deviation of 0.6 units.
- a. Describe the sampling distribution of the sample mean. [3 marks]
 - b. Find the probability that the sample mean will exceed 3.8. [3 marks]

A6 An airline deliberately overbooks seats for its flights to Europe. Past experience has shown that 90% of passengers who originally had seats booked actually turn up, (owing to cancellations, last minute illness and late arrivals at the airport). The airline takes 180 bookings for 170 available seats per flight.

Based on the assumption that passengers arrive independently, use the Normal approximation to the Binomial distribution to find:

a. the probability that for a given flight there will be unsatisfied passengers who will not have seats; [4 marks]

b. the probability that for a given flight there will be between 5 and 10 seats unfilled. [3 marks]

A7 The proportion of the time per working day that all checkouts in a particular supermarket are busy is a random variable X that takes all values x in the range $0 \leq x \leq 1$ with the probability density function (p.d.f)

$$f(x) = \begin{cases} cx^2 + x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a positive constant.

a. Show that c has to be equal to 1.5 in order for f to be a valid p.d.f. [3 marks]

b. Show that the mean and variance of X are $17/24$ and 0.048 respectively. [5 marks]

A8 A random sample of size $n = 40$ was drawn from a Normal distribution in an attempt to estimate the variability of the distribution. If the sample variance was 16.4, find the 95% confidence interval for the population variance σ^2 . [4 marks]

Section B

- B1 a. One lunchtime in MORE foodhall, there are two starters, four main course dishes and three choices for dessert.
- (i) How many choices of meal are there consisting of a starter, main course and dessert? [2 marks]
- (ii) How many choices of meal are there consisting of any two different courses? [This means that you cannot choose two starters but you could choose a starter and a dessert.] [3 marks]
- b. Members of the Brunel basketball and netball clubs join together to form a new club playing three different team games.
- Korfball is a team game in which teams consist of two women and two men.
 - Openball is a new game in which teams must consist of five players including at least two women and at least two men.
 - Cantorball is another new game in which teams consist of two players and a substitute.

The membership of the new club consists of five men and four women.

- (i) In how many different ways can the club select a Korfball team? [3 marks]
- (ii) In how many different ways can the club select a Openball team? [4 marks]
- (iii) In how many different ways can the club select a Cantorball team? [Note that for a Cantorball team you must choose two players and then choose a third player as a substitute.] [3 marks]
- c. Use induction to prove that for any $n \in \mathbb{N}$,

$$\sum_{i=0}^n (2i^2 + i) = \frac{1}{6}n(n+1)(4n+5).$$

[10 marks]

- B2 a. Michael is a cricketer with a recurring knee injury. Each time the injury returns, the number of Test matches that he misses before he is fit, is a random variable having the Binomial distribution with parameters $n = 6$ and $p = 0.4$. Suppose that the injury returns.
- (i) Find the probability that he misses exactly two matches before he is fit. [3 marks]
 - (ii) Find the probability that he misses at most two matches before he is fit. [3 marks]
 - (iii) What is the probability that he misses two matches before he is fit given that he misses at most two matches? [3 marks]
- b. Freddie owns a shop selling suits. The number of suits purchased by a customer who enters the shop is a random variable which has the Poisson distribution with parameter 0.4.
- (i) Find the probability that a randomly chosen customer buys two suits. [3 marks]
 - (ii) What is the probability that a randomly chosen customer buys at least two suits? [4 marks]
 - (iii) What is the distribution of the total number of suits bought by the first ten customers to visit the shop? [2 marks]
 - (iv) Find the probability that both of the first two customers to visit the shop do not buy a suit. [3 marks]
 - (v) Suppose that it is known that the number of customers visiting Freddie's shop between 9.00 and 10.00 has the Binomial distribution with parameters $n = 2$ and $p = 0.5$. Find the probability that on a given morning he does not sell a suit between 9.00 and 10.00. [4 marks]

- B3 a. An internal auditor of a company has reported on an investigation of a sample of 80 accounts that are overdue (they have payments outstanding). It has been found that these accounts have a mean overdue time of 53.4 days with a standard deviation of 24.8 days, and the distribution of the overdue time is positively skewed;
- (i) Calculate the 95% confidence interval for the mean overdue time for all accounts. [4 marks]
 - (ii) Explain why it is permissible to use the Normal distribution in the calculation of the confidence interval even though the distribution of the length of time accounts are overdue is positively skewed. [3 marks]
- b. In order to reduce the number of accounts overdue, the company decides to offer a 1% discount on all invoices paid within 30 days of the date of the invoice. A sample of 250 invoices sent out before the discount incentive was introduced was examined, and 42 of them had been settled within 30 days. After the new scheme had been introduced 48 invoices out of a sample of 200 were paid within 30 days.
- (i) Do these results provide enough evidence to conclude that the proportion of invoices paid within 30 days has been increased as a result of the incentive of 1% discount? [13 marks]
 - (ii) Depending on your answer to part b(i), calculate a confidence interval for EITHER the proportion of all invoices paid within 30 days, OR the proportion of invoices paid within 30 days when the 1% discount is offered. [5 marks]

B4 A customer is deciding whether to purchase a new set of brand A or brand B golf clubs. He can get a good deal on brand A clubs, but brand B are a lot more expensive, although they could improve his game. A salesperson allows the customer to try a number 3 wood at the driving range to compare their performance. The customer tries each club 15 times. The mean and standard deviation of the distances achieved by golf club brand A were 255 and 8.7 metres respectively. The distances in metres achieved by golf club brand B were as follows:

247.26 281.50 265.08 275.03 278.21 278.85 276.58 270.88
275.99 289.23 260.80 280.01 266.00 271.76 252.74

- a. (i) Obtain the mean and standard deviation of the distances achieved by golf club brand B. *You may use your calculator function.* [4 marks]
- (ii) Do these results provide evidence that the performance of brand B clubs are better than the performance of brand A clubs? [11 marks]
- b. (i) Is there evidence at the 5% significance level of a difference in the variances of the distances achieved by brand B and brand A clubs. [8 marks]
- (ii) What are the implications, if any, of your results in (b)(i) for the test carried out in (a)(i) for the difference of the two means? [2 marks]