

1. TYPE OF DEGREE: BSc.
2. SESSION: Summer 2008.
3. MODULE CODE: MA1972.
4. MODULE TITLE: Discrete Mathematics, Probability & Statistics.
5. TIME ALLOWED: 2 hours plus 5 minutes reading.
6. a. Answer all questions from Sections A
Answer two questions from section B. If more than 2 questions from section B are answered then the best two answers will be counted.
Section A carries 50% of the marks for the paper.
All questions in section B are worth equal marks.

40% needed in both sections??

7. ADDITIONAL INFORMATION: Neave statistical tables are provided.
Sue: anything else?

Section A

A1 Determine $|\{x \in \mathbb{Z}: 0 < \sqrt{x} \leq 3\}|$. [3 marks]

A2 Prove by induction that $1 + 2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 1$ for every $n \in \mathbb{N}$. [5 marks]

A3 Two dice are thrown. Let A be the event of an odd score on the first die, B be the event of an even score on the second, and C the event that either both are odd or both are even. Show that these events are pairwise independent but not collectively independent. [10 marks]

A4 For $X \sim B(5, 1/4)$ find

(i) $E(X)$; (ii) $\text{Var}(X)$; (iii) $E(X^2)$; (iv) $P(X = 3)$; (v) $P(X \geq 4)$. [7 marks]

A5 Given the following data: 32, 33, 22, 28, 24, 23, 27, 24, 27, 21, compute:

a. The mean [2 marks]

b. The standard deviation [4 marks]

A6 Let X be a random variable having expectation μ and variance σ^2 . Find the expectation and variance of the random variable nX , where n is a constant. [3 marks]

A7 A continuous random variable X takes all values x in the range $x \geq 100$ with the probability density function (p.d.f)

$$f(x) = \begin{cases} c/x^4, & x \geq 100 \\ 0, & x < 100 \end{cases}$$

where c is a positive constant.

a. Determine the constant c for $f(x)$ to be a valid p.d.f. [2 marks]

b. Using the value of c from (a), show that the mean and variance of X are 150 and 7500 respectively. [6 marks]

A8 Of a random sample of 100 students, 59 were found to be in debt. Estimate the proportion of the students in the university who were in debt with a 95% confidence interval. [3 marks]

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A9 The IQ scores of 15 individuals selected at random from an urban area in a certain country are as follows;

91 102 95 88 104 127 96 111 129 118 106 124 105 112 96

The IQ scores for the population of the whole country are known to have a standard deviation of 16. Test, at the 5% level of significance, whether or not the variation of IQ scores for the population in the urban area differs from that for the population of the country as a whole.

State any assumptions that you make.

[5 marks]

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Section B

- B1 a. Three cards are placed into a hat. One is red on both sides, the other is blue on both sides and the third is red on one side and blue on the other. A card is picked out at random and you see that one side is blue. What is the probability that the other side is also blue? [3 marks]

- b. A particular professional exam has three pass grades, A , B and C , and a single failure grade, F .

The probability of a randomly selected student scoring an A is 20%; the probability of scoring *at least* a B is 38%; and the probability of passing is 65%. Determine:

- (i) The probability that the student failed.
- (ii) The probability that the student scored a grade B .
- (iii) The probability that the student scored a grade C .

[7 marks]

- c. 45% of students attend extra revision classes and as a result the probability that a student randomly selected from this group scores a grade A increases to 30%.

- (i) What is the probability that a student randomly selected from among those who do not attend the revision classes scores a grade A ?
- (ii) What is the probability that a student who scored an A attended the revision class?

[7 marks]

- d. If the probability that a trainee accountant will pass the professional exam on any attempt is 40%, and independent of all previous attempts, what is the probability that a candidate will pass the exam:

- on the third try?
- before the third try?

What is the average number of attempts before a pass is obtained? [8 marks]

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- B2
- a. A small company employs thirty people and has premises consisting of three unequal-sized rooms. The first room can hold eight people, the second can hold fifteen people and the third can hold seven people. How many ways can the thirty employees be spread amongst the rooms? [4 marks]
- b. Once the employees have been allocated to a room, each room has to choose two team leaders. How many groups of six team leaders are possible? [3 marks]
- c. The company's computer system requires a password composed of six characters drawn from the alphabet $\{A, B, \dots, Z, a, b, \dots, z, 0, 1, 2, \dots, 9\}$. How many different passwords are possible?
- Now suppose that four of the characters in the password must be taken from $\{A, B, \dots, Z, a, b, \dots, z\}$ and the remaining two characters must be from $\{0, 1, 2, \dots, 9\}$. If no character of the same case and no number can be used twice how many passwords are possible now? [10 marks]
- d. At the firm's annual outing a thirty seater coach is hired to transport the employees. The coach has fifteen double seats (not including the driver's).
- How many ways can the employees be paired up for the journey?
 - How many ways can these fifteen pairs be seated on the coach?
- [8 marks]

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- B3 a. A paint manufacturer has developed a new type of transparent lacquer that may be used over the paintwork on cars. They wish to estimate the added protection against rust that this coating provides. A random sample of 256 pieces of steel coated with the standard car paint are tested and found to have an average life of 480 days with a standard deviation of 80 days. In addition, a random sample of 50 pieces of steel coated with both the paint and also the transparent lacquer are found to have an average life of 730 days with standard deviation of 100 days.
- Perform an appropriate test to see if there is any real evidence that the new lacquer provides added protection against rust. [11 marks]

- b. Four holiday coach tour companies all carried 600 holiday-makers during 2007 on weekend coach trips to Paris. They reported that the following numbers of customers expressed satisfaction with their coach trip.

Company	No. of satisfied customers
A	576
B	558
C	580
D	546

Use an appropriate test to see if there are significant differences in customer satisfaction between the four coach tour companies. [14 marks]

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- B4 a. From a survey of 20 randomly selected British companies it was found that the average annual expenditure on research and development (R & D) is £3.7m with a standard deviation of £0.6m. A survey of 15 similar randomly selected American companies found an equivalent average expenditure on research and development of £4.2m with a standard deviation of £0.9m.

Does this provide evidence to support the view often expressed that Britain invests less than America in R & D?

[13 marks]

- b. (i) Is there evidence at the 5% significance level, of a difference in the variances of R & D expenditure between the two countries?

[10 marks]

- (ii) What are the implications, if any, of your result in (b)(i) for the test carried out in part (a) on the differences of the two means?

[2 marks]