

MA 1920 Linear Algebra Class Test

November 2006

There are a total of 50 marks.

There are 6 questions on three pages. Answer ALL questions.

Write your solutions in the booklet provided.

Working should be shown.

1. (10 marks) Solve the following system of linear equations. Show your working. You may use any method.

$$\begin{aligned}3x & - 4z = 1 \\x + y + z & = 4 \\x - y - 2z & = 0\end{aligned}$$

2. (5 marks) For a particular system of linear equations, the augmented matrix has been transformed by elementary row operations to:

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

If the unknowns are x, y, z, t in that order, write down the most general solution for the system of equations.

3. (5 marks) For a particular system of linear equations, the augmented matrix has been transformed by elementary row operations to:

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

If the unknowns are x, y, z, t in that order, write down the most general solution for the system of equations.

4. (a) (6 marks) Evaluate the following determinant. You may use any method. Show your working.

$$\begin{vmatrix} 1 & 2 & 0 \\ 1 & -2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

- (b) (2 marks) If the determinant of a matrix is zero, what do we know about the inverse of the matrix?

5. Let $A = \begin{pmatrix} 3 & -5 \\ -2 & 4 \end{pmatrix}$.

- (a) (6 marks) Find A^{-1} . You may use any method. Show your working.

- (b) (6 marks) Check your answer to 5(a) by computing the product $A^{-1}A$ (take A^{-1} to be your solution to 5(a)). Was your answer to 5(a) correct? (Be honest here.)

6. (a) (6 marks) Let $M = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 4 & 0 \\ 2 & -1 & -1 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$. Compute the product $M\underline{v}$ or say why it doesn't exist.

(b) (4 marks) Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix. Does the product AB exist? If so, what is the order of AB ?