A1. (a) 4.

(b) 16.

A2. (a) $\frac{\sin(1/x)}{x^2}$.

(b)

$$g'(x) = 3\cos(3x). \tag{1}$$
$$g''(x) = -9\sin(3x). \tag{2}$$
$$g'''(x) = -27\cos(3x). \tag{3}$$

A3. $x \in (-\infty, -1/3) \cup (1, \infty)$.

A4. It is a circle with centre $(-3/2, 2)$ and radius 2. (Sketch omitted.)

A5. $[2, 11) \cup (11, \infty)$.

A6. (a)

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3, \\ 3-x & \text{if } x < 3. \end{cases}$$

(b) 5.

A7. (a) Diverges.

(b) Converges.

A8. Chain Rule:

$$\frac{\partial F}{\partial r} = 4r^3 \cos\theta \sin\theta.$$
$$\frac{\partial F}{\partial \theta} = r^3(\cos^2\theta - \sin^2\theta).$$

A9. The Lagrange polynomials are

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0.2)(x-0.5)}{(-0.2-0.2)(-0.2-0.5)} = \frac{(x-0.2)(x-0.5)}{0.28}.$$
$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-(-0.2))(x-0.5)}{(0.2-(-0.2))(0.2-0.5)} = \frac{(x+0.2)(x-0.5)}{-0.12}.$$
$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-(-0.2))(x-0.2)}{(0.5-(-0.2))(0.5-0.2)} = \frac{(x+0.2)(x-0.2)}{0.21}.$$

(4 marks)

$f(0.0) \approx 0.688095$.

A10. Estimate is -0.44366657. Bound on error is 0.02196.

B1. (a) i. $\ln 3$.

ii. $\frac{8}{3} - \sqrt{3}$.

(b) $x + \frac{9}{2}\ln|x-3| - \frac{1}{2}\ln|x-1| + C$.

(c) i. $1 - \frac{3}{e^2}$.

ii.

iii. $-\frac{8}{e^2} + 3I_2 = 6 - \frac{38}{e^2}$.

(d) 1.

B2. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2}{7}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\dfrac{76}{343}$.

(b) i. $g(\pi) = -\pi/2$ and $g(2\pi) = \pi/2$.

ii. $-\frac{\sin x}{|\sin x|}$.

iii. ± 1.

iv. $x \in ((2n-1)\pi, 2n\pi)$ for some integer n.

(c) 0.

(d) The point $(2, 27)$ is a local maximum and the point $(4, 23)$ is a local minimum. f is increasing on $(-\infty, 2]$ and $[4, \infty)$ but decreasing on $[2, 4]$.

B3. (a) i.

ii. 8.8125.

(b) $y = A\cos(3x) + B\sin(3x) + \frac{x}{3} + \frac{1}{5}\cos(2x)$.

B4. (a)

```
function yN=ImpEuler(a,b,y0,N)
h=(b-a)/N;
y=y0;
for i=1:N
 fa=f(a+(i-1)*h,y);
 yt=y+h*fa;
 fb=f(a+i*h,yt);
 y=y+h/2*(fa+fb);
end
yN=y;
end
function v=f(t,y)
v=t^2*exp(-t)-2*y;
end
```

(b) When $h = 1$, the estimate is 0.18393972 and when $h = 0.5$ the estimate is 0.11092394.

(c) The exact value in 1 is $y(1) = e^{-1} - 2e^{-2} = 0.09720887$.
For $h = 1$ we have the error

$$y(1) - y_1 = 0.09720887 - 0.18393972 = -0.08673084.$$

For $h = 0.5$ we have the error

$$y(1) - y_2 = 0.09720887 - 0.11092394 = -0.01371506.$$

We observe that the error has decreased to approximately 16% of the first value when we have halved the step size, this is even better then the 25% given by the theory (see d.).

(d) Improved Euler's method is $\mathcal{O}(h^2)$, i.e. we have order two.