Answers

1. TYPE OF DEGREE: BSc.

2. SESSION: May 2008.

3. MODULE CODE: MA1915.

4. MODULE TITLE: Calculus and Numerical Methods.

5. TIME ALLOWED: 3 hours (plus 5 minutes reading time).

6. a. NUMBER OF QUESTIONS: Part A has 7 questions, Part B has 4 questions. Full Marks: 100.

   b. Answer **all** questions from **Section A**. Answer **two** questions from **Section B**. If more than **two** questions from **Section B** are answered, marks from the best **two** answers will be counted.

   Section A carries 50 % of the total marks available for the paper.

   All questions in **Section B** carry equal marks.

   An indication of the marks allocated to each sub-section of a question is shown in brackets in the righthand margin.

7. ADDITIONAL INFORMATION: Calculators: Casio fx 82, Casio fx 83 and Casio fx 85 ONLY.

A1. a.

$$\frac{5}{3},$$

[3 marks]

b.

$$\frac{4}{9}.$$

[4 marks]

A2. a. (i) $f'(x) = x^2(3\ln(2x) + 1),$ [2 marks]

(ii) $g'(x) = -(12x^3 - 2)\sin(3x^4 - 2x),$ [2 marks]

b. (i)

$$R \setminus \left\{ \frac{3\pi}{2} + 2k\pi \ : \ k \in Z \right\},$$

[2 marks]

(ii)

$$h'(x) = -\frac{\cos x}{2(1 + \sin x)^{3/2}}.$$

[2 marks]

A3. a. (i)

$$\ln 2,$$

[3 marks]

(ii)

$$\frac{1}{9}(11e^3 - 8),$$

[3 marks]

b.

$$2\left(1 - \frac{1}{e}\right).$$

[3 marks]

A4. a. divergent by limit comparison test, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$, [3 marks]

b. convergent by limit comparison test, compare with $\sum_{n=1}^{\infty} e^{-n}$. [3 marks]

A5.

$$e^{-x^2} = 1 - x^2 + \frac{1}{2}x^4 \pm \cdots,$$

$$\frac{443}{960} \approx 0.46146,$$

$$0.5.$$

A6. a. $f(0) = -\ln 2 < 0$, $f(1) = 1 > 0$, use intermediate value theorem, [2 marks]

b. $x \approx 0.52501$.

A7. $f(0.5) \approx 1.1375$. [7 marks]

B1. a. $f(x) = (\sin x - 2)(\sin x - 1)$, $f(0) = f(\pi) = f(2\pi) = 2$. [4 marks]

b. $f'(x) = (2\sin x - 3)\cos x$
$\left(\frac{\pi}{2}, 0\right)$, local min. and absolute min.,
$\left(\frac{3\pi}{2}, 6\right)$, local max. and absolute max., [5 marks]

c. $f''(x) = 2 - 4\sin^2 x + 3\sin x$,
$(3.58099, 0.90479)$, $(5.84379, 0.90479)$, both points of inflection. [5 marks]

d. graph not in this file. [4 marks]

e.

$$g \circ f \,:\, \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 2\pi\right] \to R, \quad (g \circ f)(x) = \ln(\sin^2 x - 3\sin x + 2)$$

$\left(\frac{3\pi}{2}, \ln 6\right)$ absolute maximum,
vertical asymptote at $y = \frac{\pi}{2}$, no absolute minimum,
no horizontal asymptotes,
graph not in this file. [7 marks]

B2. a.

$$y = (x+1)e^{x^2},$$

[6 marks]

b.

$$y = (Ax + B)e^{-3x} + \frac{10}{169}\cos(2x) + \frac{24}{169}\sin(2x),$$

[12 marks]

c.

$$\frac{dy}{dx} = \frac{4}{2-\pi}, \quad y = \frac{4x}{2-\pi} - \frac{\pi^2}{4(2-\pi)}.$$

[7 marks]

B3. a. at $(0,0)$ saddle point,
at $(1,1)$ local minimum,
other two: $(0,2)$, $(-1,1)$, [13 marks]

b. normal vector $(-3,-6,1)^T$,
tangent plane $z = 3x + 6y + 2$, [6 marks]

c. 16. [6 marks]

B4. a. (i)
```
function [t y]=Euler(t0,y0,b,n)
h=(b-t0)/n;
y=size(n+1,1); t=size(n+1,1);
t(1)=t0; y(1)=y0;
for i=1:n
 t(i+1)=t0+i*h;
 y(i+1)=y(i)+h*f(t(i),y(i));
end
end
function v=f(t,y)
v=t*exp(-3*t)-y
end
```
[10 marks]

(ii) $h = 0.1$: $y(0.2) \approx 0.0074081822$,
$h = 0.05$: $y(0.2) \approx 0.0102430700$, [10 marks]

(iii) $h = 0.1$: exact value $y(0.2) = 0.012598618$, error 0.0051986
$h = 0.05$: exact value $y(0.2) = 0.012598618$, error 0.00236.
The error has decreased by approximately 50% when we have halved the step size. [3 marks]

(iv) Euler's method is $O(h^1)$, i.e. order of error is one. [2 marks]