Answers

1. TYPE OF DEGREE: BSc.

2. SESSION: May 2007.

3. MODULE CODE: MA1915.

4. MODULE TITLE: Calculus and Numerical Methods.

5. TIME ALLOWED: 3 hours (plus 5 minutes reading time).

6. a. NUMBER OF QUESTIONS: Part A has 7 questions, Part B has 4 questions. Full Marks: 100.

   b. Answer **all** questions from **Section A**. Answer **two** questions from **Section B**. If more than **two** questions from **Section B** are answered, marks from the best **two** answers will be counted.

   Section A carries 50 % of the total marks available for the paper.

   All questions in **Section B** carry equal marks.

   An indication of the marks allocated to each sub-section of a question is shown in brackets in the righthand margin.

7. ADDITIONAL INFORMATION: Calculators: Casio fx 82, Casio fx 83 and Casio fx 85 ONLY.

A1. a.

$$-\frac{1}{2}$$

[3 marks]

b.

$$-\frac{3}{2}.$$

[3 marks]

A2. a. (i) $f'(x) = (3x^2 + 2x^3)e^{2x},$ [2 marks]

(ii) $g'(x) = 5\cos(x^5)x^4.$ [2 marks]

b. (i)

$$h'(x) = \frac{1}{2+x} + \frac{1}{2-x}.$$

[2 marks]

(ii)

$$(-2, 2)$$

[2 marks]

A3. a. (i)

$$\frac{1}{4}$$

[3 marks]

(ii)

$$\frac{2}{9}e^{-3} - \frac{5}{9}$$

[3 marks]

b.

$$-\frac{2}{\sqrt{u}} - \ln|u| - \frac{1}{u} + C$$

[3 marks]

A4.

$$y = \frac{3}{2}x - \frac{3}{4} - \frac{3}{4}e^{2(1-x)}$$

[6 marks]

A5. $(0, 0)$ is a saddle point. $(\frac{1}{2}, \frac{1}{4})$ is a local minimum. [8 marks]

A6. $y(1.1) \approx 1.702$, correct to 3 decimal places. [6 marks]

A7. $f(1.5) \approx 1.4375$. [7 marks]

B1. a. $g(x) = (x-4)(x-1)$, natural domain of $f = R \setminus \{1, 4\}$, $x = 0$. [3 marks]

b.

$$\lim_{x\to\infty} f(x) = 0, \lim_{x\to-\infty} f(x) = 0.$$

[2 marks]

c. Vertical asymptotes: $x = 4$ or $x = 1$. Horizontal asymptotes: $y = 0$. [3 marks]

d. $(2, -1)$ local maximum, $(-2, -\frac{1}{9})$ local minimum. No absolute maximum or minimum. [5 marks]

e. Solution exists by the intermediate value theorem. [3 marks]

f. For graph see extra file. [5 marks]

g.

$$2\ln 2 - \frac{1}{3}\ln 5$$

[4 marks]

B2. a.

$$y = x(3\ln|x| + 1)^{1/3}$$

[9 marks]

b.

$$y = \frac{-39 + 19e}{25(e - e^5)}e^{5x} + \frac{39 - 19e^5}{25(e - e^5)}e^x + \frac{x}{5} + \frac{6}{25}$$

[12 marks]

c. convergent geometric progression, $\frac{108}{7}$ [4 marks]

B3. a.

$$f(x, y) = x + 3x^2 + \cdots$$

[10 marks]

b. $(-1, 0, 1)$. [3 marks]

c. $\left(-\frac{12}{11}, -\frac{7}{11}\right)$ is a local minimum point (at $\lambda = \frac{11}{4}$).

$\left(\frac{12}{11}, \frac{7}{11}\right)$ is a local maximum point (at $\lambda = -\frac{11}{4}$). [12 marks]

B4. a. (i) intermediate value theorem [2 marks]

(ii) For graph see extra file. One root. [6 marks]

b. (i)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

[2 marks]

(ii) $X = 0.53978516 \pm 6.7 \times 10^{-7}$ [10 marks]

c. (i)

$$\int_{x_1}^{x_n} f(x)\,dx \approx \frac{h}{6} \sum_{i=1}^{n-1} \left[f(x_i) + 4f\left(x_i + \frac{h}{2}\right) + f(x_{i+1}) \right],$$

where $|x_{i+1} - x_i| = h$. [2 marks]

(ii) -0.0353 correct to 4 decimal places. [3 marks]