Section A

A1. a. The series converges to 8.

b. $\frac{11}{34}$.

A2. a. $\frac{3}{2}$.

b. $-\frac{1}{2}$.

A3. a.

b. $\frac{1}{1+x^2}$.

c. 1.

A4. a. 1.

b. $\frac{1}{4}$.

A5. a.

$$|x-2| = \begin{cases} x-2, & \text{if } x \geq 2, \\ 2-x, & \text{if } x < 2. \end{cases}$$

b. $\frac{5}{2}$.

A6. The second critical point is at $(2,1)$. The point $(0,0)$ is a saddle point and the point $(2,1)$ is a local minimum.

A7. The solution is $X = 0.875 \pm 0.125$.

A8. The polynomial is

$$P(x) = 1.76(3-10x) + 1.58(10x-2).$$

$f(0.4) \simeq 1.4$.

Section B

B1. a. $f(x) = (x-1)(x+1)(x-3)$. So the roots of $f(x)=0$ are $-1, 1, 3$.

b.

$$\text{dom}(f) = \mathbb{R}, \quad \text{codom}(f) = \mathbb{R}, \quad \text{dom}(g) = \mathbb{R}\backslash\{-1,1,3\}, \quad \text{codom}(g) = \mathbb{R}.$$

c. There is a local maximum at $1 - \frac{2\sqrt{3}}{3}$ and a local minimum at $1 + \frac{2\sqrt{3}}{3}$.

d. $f''(x) = 0$ when $x = 1$. This is a point of inflection.

e. g has vertical asymptotes at $x = -1, 1, 3$. It has a horizontal asymptote at $y = 0$.

f. 0. This means that between -1 and 3 the area below the x-axis and above the graph equals the area above the x-axis and below the graph.

B2. a. $y = x - 1 + 2e^x$.

b. $y = x\sqrt{\ln x^2 + 1}$.

c. $y = \frac{e^{3(1-x)}}{1-e^2} - \frac{e^{3-x}}{1-e^2} + \frac{1}{3}x - \frac{4}{9}$.

B3. a. $ye^{2xy} = 1 + 2x + (y-1) + 2x^2 + 4x(y-1) + \cdots$.

b. The critical points are at $\left(\frac{3}{5}, \frac{4}{5}\right)$ and $\left(-\frac{3}{5}, -\frac{4}{5}\right)$. The first is a maximum with value 5 and the second is a minimum with value -5.

B4. a.

b. When $h = 0.1$, an estimate for $f(0.3)$ is 0.0048.
When $h = 0.05$, an estimate for $f(0.3)$ is 0.0063.

c. It is an explicit formula.

d. (i) The error approximately halves when h is halved.

(ii) We would expect an error of $O(h)$.