

**Department of Electronics & Computer Engineering**

**Mid-Session Test January 2002**

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**EE3052B - MultiMedia Signal Processing**

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**Time allowed 1 Hours**

**Answer *three* out of five questions**

Ensure that your registration number is written clearly on the front cover.

Answers

**Q1.**

(a) where  $n$  is an integer

(a)

$$\begin{aligned} X(k + n * N) &= \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}(k+n*N)m} = \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}km} e^{-j2\pi n*m} \\ &= \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}km} = X(k) \end{aligned}$$

(3 Marks)

(b) The main assumption in DFT is that the signal is periodic in time and hence its frequency spectrum is discrete.

(1 Mark)

(c) The signal is windowed to reduce the end point discontinuity and the consequent spread of energy to higher frequencies.

(1 Mark)

(d) Hamming or Hanning windows:

Raised cosine window

$$w(m) = \alpha - (1 - \alpha) \cos(2\pi m / N - 1)$$

Hamming  $\alpha = 0.54$  or Hanning  $\alpha = 0.5$ .

(2 Marks)

(e) Time resolution is the length of time window in seconds. Ideally time resolution should be set less than the duration of smallest event in the signal. Frequency resolution is the distance in frequency between successive DFT samples.

(1 Mark)

(f)  $\Delta f = 1/\Delta T = 1/NT_s = 1/N$   
or  $\Delta f \Delta T = 1$

The Uncertainty Principle states that the product of time resolution and frequency resolution is a constant, hence one can only be increased or decreased at the expense of the other.

(2 Marks)

**Q. 2**

(a)

$$\begin{aligned} \text{(i)} \quad X(0) &= x(0) + x(1) \\ X(1) &= x(0) + e^{-j\pi} x(1) \end{aligned}$$

(2 Marks)

$$\begin{aligned} \text{(ii)} \quad X_{padded}(0) &= x(0) + x(1) \\ X_{padded}(1) &= x(0) + e^{-j\pi/2} x(1) \\ X_{padded}(2) &= x(0) + e^{-j\pi} x(1) \\ X_{padded}(3) &= x(0) + e^{-j3\pi/2} x(1) \end{aligned}$$

(3 Marks)

(b) A DFT is used as part of a digital signal processing system for the analysis of an analog music signal with significant frequency content of up to 20 kHz. Calculate:

$$\text{(i)} \quad F_s = 40 \text{ kHz and } 44.100 \text{ kHz.}$$

(1 Mark)

$$\text{(ii)} \quad \Delta f = 1/\Delta T = 1/NT_s = F_s/N$$

$$10 > 44100/N$$

$$N > 4410 \text{ samples}$$

(2 Marks)

(iii) Improvement in the actual frequency resolution resulting from zero-padding is zero.

Improvement in apparent resolution is a factor of 2.

(2 Marks)

**Q.3**

(a) Taking the z-transform yields

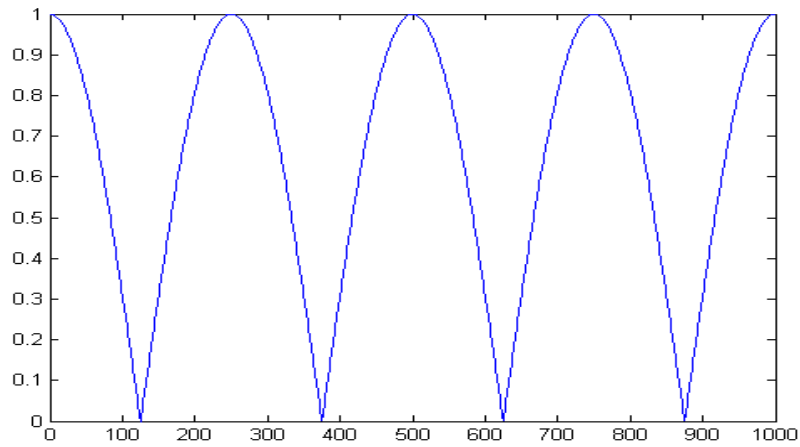
$$X(z) = z^{+k} + z^{-k}$$

(2 Marks)

To obtain the frequency response substitute  $z = e^{j\omega}$

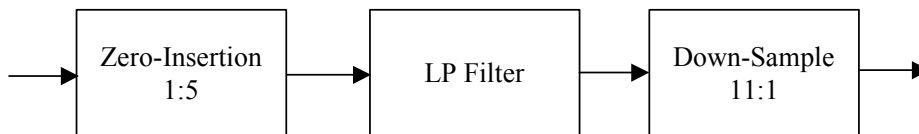
$$X(e^{j\omega}) = e^{jk\omega} + e^{-jk\omega} = 2 \cos(k\omega)$$

The following figure shows the frequency response for  $k=2$ .



(2 Marks)

- (b) For sampling rate conversion first insert four zeros between every two samples to increase the sampling rate by a factor of 5, then low pass filter the signal to  $1/11^{\text{th}}$  of the half sampling frequency and then down sample by a factor of 11:1.



(5 Marks)

The main effect of down sampling will be that the spectrum above 5 kHz will be lost.

(1 Mark)

## Q.4

- (a) (i) The linear difference equation describing the input output relation in time of the system expressed in the following Equation

$$y(m) = \frac{1}{a_0} (a_1 y(m-1) + a_2 y(m-2) + b_0 x(m) - b_1 x(m-1) - b_2 x(m-2))$$

(2 marks)

- (ii) Express the transfer function of the second order system of Equation (3) in the polar form.

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{b_0 - b_1 z^{-1} - b_2 z^{-2}}{a_0 - a_1 z^{-1} - a_2 z^{-2}} = g \frac{(1 - z_1 z^{-1})(1 - z_1^* z^{-1})}{(1 - p_1 z^{-1})(1 - p_1^* z^{-1})} \\ &= g \frac{1 - 2r_z \cos \varphi_z z^{-1} + r_z^2 z^{-2}}{1 - 2r_p \cos \varphi_p z^{-1} + r_p^2 z^{-2}} \end{aligned}$$

(2 marks)

- (b) (i)  $F_{\text{notch}} = 5 \text{ kHz}$ .

In normalised frequency terms

$$\varphi_{\text{notch}} = 2\pi F_{\text{notch}} / F_s = \pi / 2$$

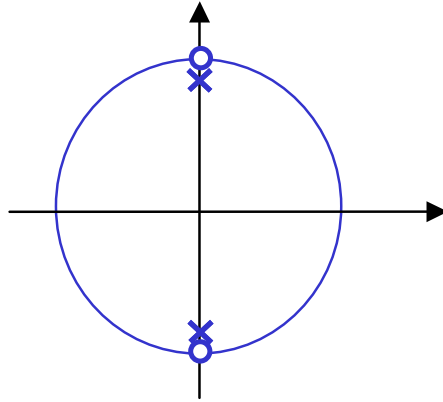
for a notch filter the pole radius  $r_z = 1$ .

Hence  $H(z)$  becomes

$$H(z) = \frac{Y(z)}{X(z)} = g \frac{1 + z^{-2}}{1 + r_p^2 z^{-2}}$$

In angular terms then zeros are at  $z = +j$  and  $z = -j$ , the poles are at  $z = jr_p$  and  $z = -jr_p$ .

(4 marks)



(1 Mark)

As the pole radius  $r_p$  approaches 1 the notch bandwidth decreases and the notch becomes more frequency selective and its time constant increase or decrease with the pole radius..

(1 marks)

**Q.5**

(a)

Normalised cutoff frequency in radians  $=2\pi f/F_s=2\pi*2000/20000$

$$=\pi/5.$$

Cutoff frequency a a fraction of Sampling Frequency  $=f/F_s=0.1$

Using the window design technique the FIR impulse response is obtained from the inverse Fourier transform as

$$h_d(m)=\int_{-0.1}^{0.1} 1.0 e^{j2m\pi f} df$$

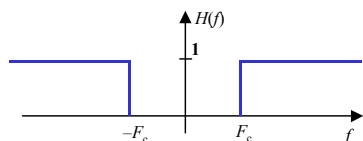
we obtain the FIR filter response as

$$h_1(m)=w(m)\times 0.2\text{sinc}(0.2\pi(m-M/2)) \quad 0 \leq m \leq M$$

Note for causality the filter impulse response is windowed and delayed.

[4 marks]

Consider the design of a high-pass linear-phase FIR filter with a frequency response given by



$$H_d(f) = \begin{cases} 0 & |f| < F_c \\ 1.0 & \text{otherwise} \end{cases}$$

The impulse response of this filter is obtained via the inverse Fourier integral as

$$\begin{aligned} h_d(m) &= \int_{-1/2}^{-F_c} 1.0 e^{-j2m\pi f} df + \int_{F_c}^{1/2} 1.0 e^{-j2m\pi f} df = \frac{e^{-j2m\pi f}}{-j2m\pi} \Big|_{-1/2}^{-F_c} + \frac{e^{-j2m\pi f}}{-j2m\pi} \Big|_{F_c}^{1/2} \\ &= \frac{\sin \pi m}{m\pi} - \frac{\sin 2\pi F_c m}{m\pi} \end{aligned}$$

To introduce causality ( $h(k) = 0$  for  $k < 0$ ) truncated shift  $h(k)$  by  $M/2$  samples

$$h(m) = \frac{\sin \pi (m - M/2)}{(m - M/2)\pi} - \frac{\sin 2\pi F_c (m - M/2)}{(m - M/2)\pi}$$

(3 Marks)

To design the band-pass filters we can use the amplitude modulation (AM) method to convert a low-pass filter to a band-pass filter. For a band-pass width of 1 kHz the low-pass filter should have cutoff frequencies of  $\pm 500$  Hz (note that from  $-500$  to  $+500$  Hz we have a bandwidth of 1 kHz). Thus the required low pass FIR filter equation is given by

$$h(m) = w(m) \times 0.05 \text{ sinc}(0.05\pi(m - M/2)) \quad 0 \leq m \leq M$$

To translate this low pass filter to the specified band pass filters we need AM sinusoidal carriers with frequency 2.5 kHz (normalise frequency  $2.5/20 = 1/8$ ). The modulated band pass filter equations are given by

$$h_2(m) = 2 \times w(m) \times 0.05 \text{ sinc}(0.05\pi(m - M/2)) \sin(\pi m / 4) \quad 0 \leq m \leq M$$

where M is even

[3 Marks]