Department of Electronics& Computer Engineering

Mid-Session Test January 2002

EE3052B - MultiMedia Signal Processing

Time allowed 1 Hour

Answer *<u>three</u>* out of five questions

Ensure that your registration number is written clearly on the front cover.

Q1.

The discrete Fourier transform (DFT) is given by

$$X(k) = \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}mk} \qquad k = 0, \ldots, N-1 \qquad \text{Eq (1)}$$

(a) Show that the DFT spectrum is periodic along the frequency axis with a period of N.

(3 Marks)

(b) State the fundamental assumption in the DFT regarding the pattern, along the time axis, of the discrete-time signal $x(m)$, when its spectrum is sampled and thus converted to discrete frequency.

(1 Marks)

(c) Describe why signals are usually windowed in preparation for computing the DFT.

(1 Mark)

(d) Suggest two useful windows and write their equation.

(2 Marks)

(e) Describe what is meant by the time and the frequency resolutions in the DFT.

(1 Mark)

(f) Write an expression describing the relation between the time and the frequency resolutions. Hence, explain the Uncertainty Principle.

(2 Marks)

Q. 2

(a) Obtain the discrete Fourier transforms (DFT) of the signal vectors $\boldsymbol{x}$ and $\boldsymbol{x}_{\text{padded}}$, where $\boldsymbol{x}_{\text{padded}}$ is the zero-padded version of $\boldsymbol{x}$. These are given by:

(i) $\boldsymbol{x}=[x(0), x(1)]$
(ii) $\boldsymbol{x}_{\text{padded}}=[x(0), x(1), 0, 0]$.

(5 Marks)

(b) A DFT is used as part of a digital signal processing system for the analysis of an analog music signal where the music has significant frequency content of up to 20 kHz.

(i) State the minimum sampling rate F_s required, and also a music industry-standard sampling rate.

(1 Mark)

(ii) Obtain the number of time domain samples required to achieve a frequency resolution of 10 Hz at the minimum sampling rate.

(2 Marks)

(iii) Quantify the improvement in the actual frequency resolution and also the apparent frequency resolution if the N signal samples in (ii) is padded by N zeros.

(2 Marks)

Q.3

(a) Obtain an expression for the z-transform of the discrete-time impulse function given by

$$x(t) = \delta(t+k) + \delta(t-k) \qquad \text{(2 Marks)}$$

where $\delta(t)$ is the impulse function. Hence obtain and sketch the magnitude frequency spectrum of this function and suggest an application for the system.

(2 Marks)

(b) With the aid of a block diagram, describe how a signal originally sampled at 44 kHz can be down sampled to 20 kHz.

(5 Marks)

State the main quantitative effect of this down sampling process on the spectrum of the signal.

(1 Mark)

Q.4

(a) (i) Write the linear difference equation describing the input-output relation in time of the system expressed in the following equation:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 - b_1 z^{-1} - b_2 z^{-2}}{a_0 - a_1 z^{-1} - a_2 z^{-2}} \qquad \text{Eq (2)}$$

(2 Marks)

(ii) Express the transfer function of this second order system in polar form.

(2 Marks)

(b) (i) Using the polar form of Eq (2), design a notch filter operating at a sampling rate of 20 kHz and with a notch frequency of 5 kHz.

(4 Marks)

(ii) Draw the pole-zero diagram for the notch filter in (i) and describe the effect of varying the pole position on the bandwidth and the impulse response of the system.

(2 Marks)

Q.5

(a) Using the inverse Fourier transform and the window design technique, design a low pass filter to operate at a sampling frequency of 20 kHz with a cutoff frequency of 2 kHz.

(4 Marks)

(b) Convert the low pass filter of part (a) to a high pass filter with a cutoff frequency of 2 kHz.

(3 Marks)

(c) Modify and convert the lowpass filter of part (a) to a bandpass filter with a bandwidth of 1 kHz centred at 3 kHz.

(3 Marks)