UNIVERSITY OF BRADFORD

QUANTITATIVE METHODS IN FINANCE (MSc)

MAN4265M

11 January 2013 **16:00 - 17:30 hours**

This is a **CLOSED BOOK** examination

Answer a total of **THREE** questions

Answer **TWO** questions from Section A
All questions carry equal weighting.

Answer **ONE** question from Section B
All questions carry equal weighting.

Non-programmable calculators without any long-term data memory are permitted. Calculators should not be able to store any formulae.
Statistical tables are provided.

Section A (Answer TWO questions from this section)

1. (a) Find the yearly rate of return of an investment that, for an initial cost of 100, returns 110 after 2 years.

   [5% weighting]

   (b) What is the effective interest rate when the nominal interest rate of 10% is

   i. compounded semiannually;
   ii. compounded continuously?

   [(2+3)% weighting]

   (c) How much do you need to invest at the beginning of each of the next 60 months in order to have a value of $100,000 at the end of 60 months, given that the annual nominal interest rate will be fixed at 6% and will be compounded monthly?

   [5% weighting]

   (d) What is the value of the continuously compounded nominal interest rate r if the present value of 104 to be received after 1 year is the same as the present value of 110 to be received after 2 years?

   [5% weighting]

   (e) Suppose a 1-year government bond yields 7 per cent and the inflation rate is expected to be 6 per cent. What is the real interest rate? What value will you obtain if you make an approximation based on the Fisher equation?

   [5% weighting]

   [Total 25% weighting]

2. Multivariate regression.

Using data for a sample of US houses, you are provided the following regression output obtained using R:

Model 1: OLS, using observations 1–92
Dependent variable: price

	Coefficient	Std. Error	t-ratio	p-value
const	−18.6080	29.1629	−0.6381	0.5251
bdrms	11.6845	9.5687	1.2211	0.2253
sqrft	0.1301	0.0136	9.5564	0.0000
colonial	13.0232	7.4079	1.7592	0.0817

Mean dependent var	291.7179	S.D. dependent var	101.5164
R^2	0.640486	Adjusted R^2	0.628230
$F(3, 88)$	52.25824	P-value(F)	1.72e–19
Log-likelihood	−508.0418	Akaike criterion	1024.084
Schwarz criterion	1034.171	Hannan–Quinn	1028.155
N=92			

where **price** is house price in US dollars ($), **sqrft** is area of the house in square feet, **bdrms** indicates the number of bedrooms that the house has and **colonial** is a binary variable equal to 1 if the house was originally built during the colonial period. Please answer the following questions:

(a) Using the output provided, write down your estimated regression equation and comment on the significance of individual regressors. Also comment on the results for the F-test. Mention appropriate null hypotheses clearly.

[6% weighting]

(b) What is the estimated increase in price for a house with an additional bedroom where the bedroom is 150 square feet in size?

[2% weighting]

(c) The first house in the sample has an area $sqft$=2525, $bdrms$=4 and $colonial$=1. Find the predicted selling price for this house using the OLS line of best fit.

[3% weighting]

(d) The actual selling price of the first house in the sample was $400,000 (so price equals 400). Find the residual for this house. Does it suggest that the buyer underpaid or overpaid for the house? Please note - you should use some information from part (c) to answer this question.

[4% weighting]

(e) A diagnostic test in `R` provides the following output for our estimated model for house prices:

```
R> bptest(model.houseprices)
studentized Breusch-Pagan test
data:  houseprices
BP = 14.0205, p-value = 0.002877
```

What problem does this diagnostic indicate, if any (state the associated null hypothesis also)? If a problem is detected by this test, what are possible remedies? What are the consequences of ignoring this type of problem? Is this type of problem more common in cross sectional data or time series data (give reasons)?

[10% weighting]

[Total 25% weighting]

3. Time Series Analysis.

The following UK time series are analysed: `cpi`: consumer price index (1967 = 100) and `ur`: total unemployment rate (percent). Annual data is employed for both series (1900-1970). The `R` output from Augmented Dickey Fuller tests carried out on `cpi` and `ur` in levels are provided in Table 1. 3 lags are allowed for and a trend and constant is included in `cpi`, while only a constant is included for `ur`.

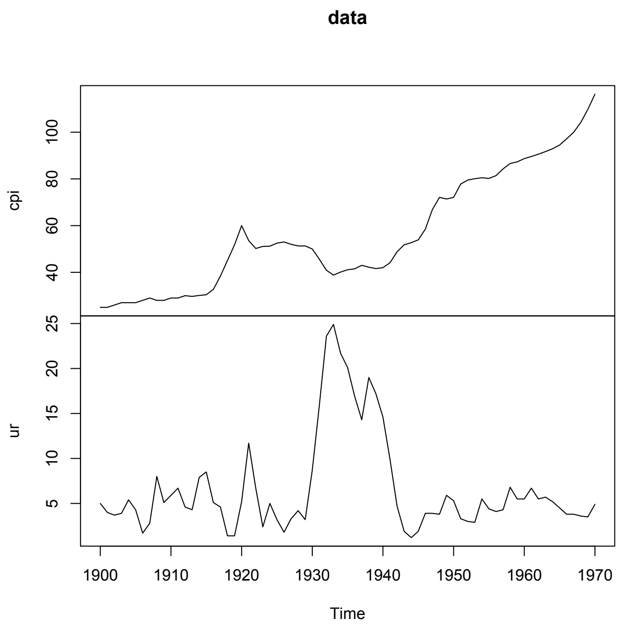

Figure 1: Plots of `cpi` (top) and `ur` (bottom)

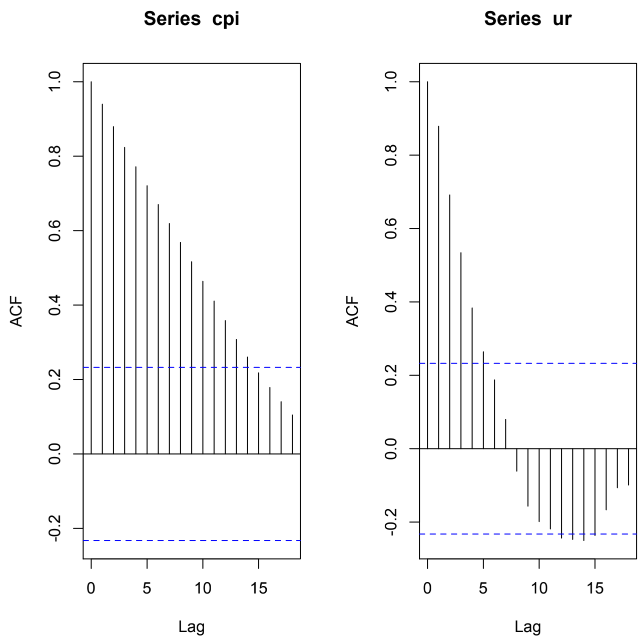

Figure 2: ACF of `cpi` (left) and `ur` (right)

```
TABLE 1

ADF Test on cpi
###############################################
# Augmented Dickey-Fuller Test #
###############################################

Value of test-statistic is: -2.2570 2.5096 3.5742

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

```
ADF Test on ur
###############################################
# Augmented Dickey-Fuller Test #
###############################################

Value of test-statistic is: -2.7632 3.950

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

(a) Based on the ADF test output provided (Table 1), what can you say about stationarity properties of `cpi` and `ur`? Please state your null hypothesis also.

[5% weighting]

(b) Define stationarity. What does the presence of a unit root imply for the assumption of mean reversion in stationary series? Comment on the time series graphs and the ACF graphs provided in Figures 1 and 2.

[6% weighting]

(c) If a time series is $I(1)$ what simple transformation would allow the series to be included in standard empirical models? Why do we need to carry out this additional analysis?

[4% weighting]

(d) Based on the `R` output for `cpi` in Table 1 above, what can you say about significance of the drift parameter?

[5% weighting]

(e) What is a random walk model? Provide a suitable example for an application of random walk models in economics or finance.

[5% weighting]

[Total 25% weighting]

Section B: Answer ONE question from this section. Please start each answer on a separate page.

4. (a) Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

[10% weighting]

(b) Experience gained over the last few years shows that for an examination on probability theory, 60% pass the written part of the examination. The written part is much harder than the oral part of the examination. 95% pass the oral part of the examination. To pass the whole examination one has to pass both of its parts and the rule is that one must pass the written part first to be allowed to take the oral examination. What is the probability that a student who fails the examination, has failed the written part? [Hint: Use the Bayes' Theorem.]

[15% weighting]

(c) The cost per unit c of producing DVD players over a two-year period is given by

$$c = 0.005t^2 + 0.01t + 13.15, \quad 0 \leq t \leq 24$$

where t is time in months. Approximate the average cost per unit over the two-year period. Will the average cost per unit be less than £15? (Hint: you need to compute a definite integral).

[10% weighting]

(d) Suppose that production function Y is given by a Cobb-Douglas production function

$$Y(K, L) = K^\alpha L^\beta$$

where K is the level of capital and L is labour input. Totally differentiate Y and calculate the percentage increase in output, given that both capital and labour increase by 20%. Assume $\alpha = \frac{1}{6}$, $\beta = \frac{5}{6}$.

[15% weighting]

[Total 50% weighting]

5. (a) i. What do you understand by ARCH and GARCH effects?

ii. Assuming that R_t denotes the time series of a stock return, set out the basic GARCH(p,q) model and show how it is linked to an ARCH(p) model. (Derivations are not expected - simply state the two model types formally.)

iii. What is the most common type of GARCH model used in practice? Briefly outline some extensions to the GARCH model.

iv. Using the `R` output provided below (Table 2), write the equation of the estimated GARCH model for stock returns and interpret this model.

TABLE 2

Error Analysis:

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.23587	0.03509	3.357	0.000486
omega	0.09870	0.01851	2.605	0.008843
alpha1	0.22343	0.03771	3.061	0.002118
beta1	0.79478	0.05422	14.842	< 2e-12

[10%+10%+5%+5% weighting]

(b) What is an Ito process? Why is the Ito process more applicable to describing stochastic processes underlying asset prices than either the basic Wiener process or the generalised Wiener process?

[10% weighting]

(c) Let P_t be the price of a stock at time t which is continuous in $[0, \infty)$. It is conventional to assume that P_t follows the special Ito process

$$dP_t = \mu P_t dt + \sigma P_t dw_t$$

where μ and σ are constant and w_t is a Wiener process. Apply Ito's lemma to obtain a continuous time model for the natural logarithm of the stock price P_t.

[10% weighting]

[Total 50% weighting]

6. Consider a three-year coupon-paying bond that pays coupons at an annual coupon rate of 4% and has a face value of 100. The current market yield to maturity on this bond is 3.5% per annum. Determine the value of the bond if the yield to maturity changes to 3.7% (please provide answers correct to four decimal places):

   (a) Using the linear Taylor series approximation.

   [10% weighting]

   (b) Using the quadratic Taylor series approximation.

   [10% weighting]

   (c) Calculate modified duration and convexity.

   [10% weighting]

   (d) Explain (non-mathematically) what is meant by the Macaulay duration. Briefly discuss why it is useful and explain one weakness it suffers from.

   [10% weighting]

   (e) Calculate the percentage change in the coupon-paying bond price as result of the increase in the yield-to-maturity using the modified Duration and convexity terms.

   [10% weighting]

[Total 50% weighting]

END OF EXAM PAPER