

DEGREE EXAMINATION

MX4540 Knot Theory

Tuesday 23 May 2006

(3 pm—5 pm)

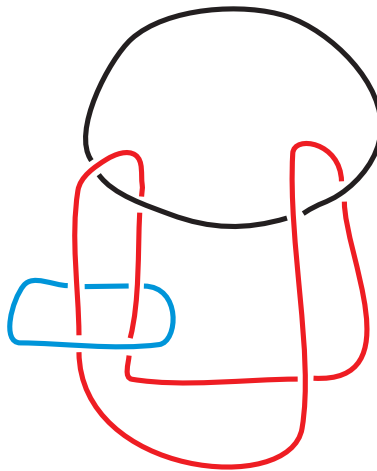
Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

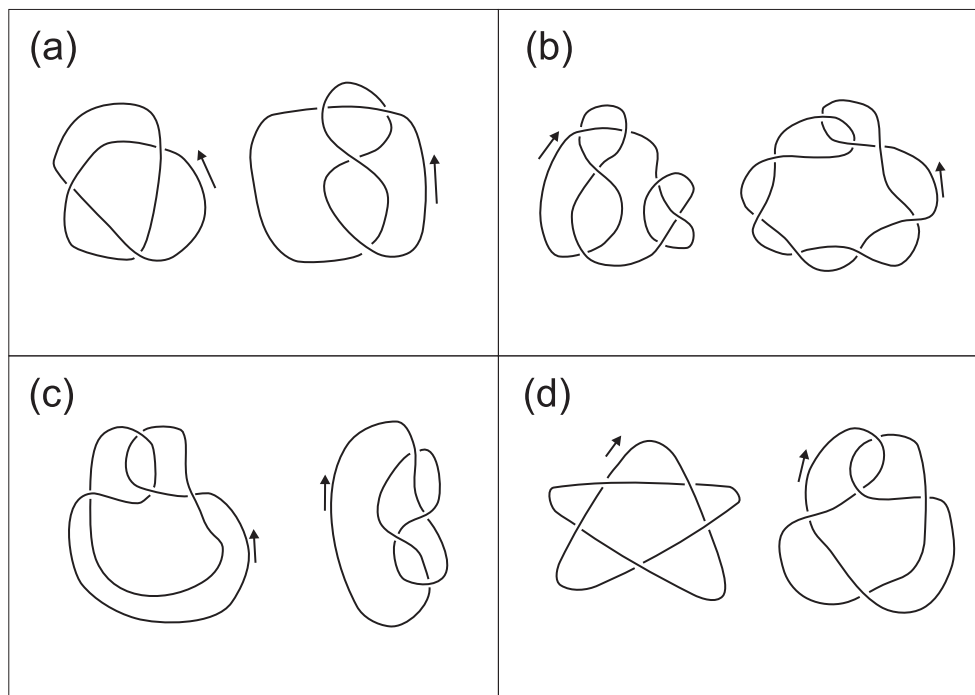
Answer THREE questions. All questions carry equal weight.

1.
 - (a) Define what it means to say that two knots K_1 and K_2 are equivalent.
 - (b) Define the Reidemeister moves and state the Reidemeister theorem.
 - (c) Define the linking number of an oriented link and prove that it is a well defined invariant of an oriented link.
 - (d) Draw a diagram of a link with linking number equal to 5.
 - (e) Prove that the Hopf link is not equivalent to the Whitehead link.

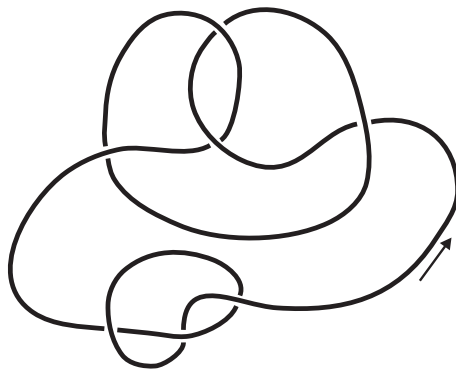
2.
 - (a) Define what it means to say that a link L can be coloured mod n .
 - (b) Define what it means to say that a link L is splittable.
 - (c) Prove that the trefoil and the figure eight knot are not equivalent.
 - (d) Draw an example of a knot together with its colouring mod 5.
 - (e) Determine if the following link is splittable.



3. Determine which pairs consist of equivalent knots. Draw relevant pictures if the knots are equivalent or give a proof when they are not.



4. Let K be the following knot.



- Define what it means to say that two oriented links are skein related.
- Determine the Alexander polynomial of K .
- Compute the determinant of K .
- Determine the genus of K .
- Determine the colouring group of K .