UNIVERSITY OF ABERDEEN **SESSION 2004–05**

DEGREE EXAMINATION

MX4540 Knots

Wednesday 25 May 2005 (3 pm—5 pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer THREE questions. All questions carry equal weight.

1. **(a)** Let $f, g : [0, 1] \to (X, p)$ be based loops. Define what it means to say that f and g are homotopic.

(b) Let $f, g : [0, 1] \to (\mathbb{R}^2, (1, 0))$ be the based loops defined by:

$$f(t) := (\cos 2\pi t, \sin 2\pi t) \qquad g(t) := (1, 0).$$

Write a formula for a homotopy between f and g.

(c) Give an example of based loops $f, g : [0, 1] \to \mathbb{R}^2 - \{(0, 0)\}$ which are not homotopic.

(d) Write down the fundamental group $\pi_1(\mathbb{R}^2 - \{(0, 0)\})$ and give a loop representing a generator of this group.

2. **(a)** State the Van Kampen theorem.

(b) What are the following groups (you may write presentations as well as use symbols, *e.g.* $\mathbb{Z} * \mathbb{Z}/7\mathbb{Z}$):

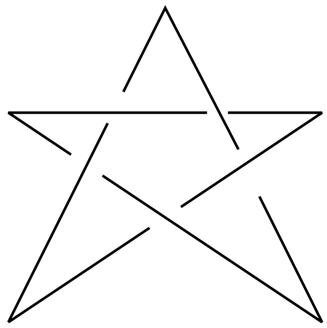
(i) $\pi_1(S^1 \vee S^3)$?

(ii) $\pi_1(T^2 \vee T^2)$?

(iii) $\pi_1(S^2 - \{p, q, r\})$, where p, q, r are distinct points on S^2?

(c) Give an example of a space X whose fundamental group is isomorphic to $\mathbb{Z}/3 * \mathbb{Z}/5$ (the free product of cyclic groups of order 3 and 5). You may draw pictures.

3. Let K be the knot in $\mathbb{R}^3$ depicted in the following diagram.



(a) Give a presentation of the fundamental group of the complement of K.

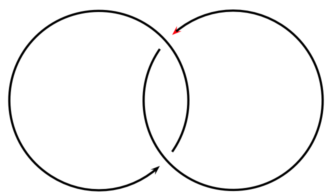
(b) Let $p, q \in \mathbb{Z}$ be relatively prime integers greater than one. Let $K_{p,q}$ be a torus knot of type (p, q). Write down the fundamental group of its complement $\mathbb{R}^3 - K_{p,q}$ and prove that it is infinite and not Abelian.

(c) Prove that the torus knots $K_{2,3}$ and $K_{2,5}$ are not equivalent.

4. **(a)** Define what it means to say that three oriented links L_-, L_0 and L_+ are skein related.

(b) Define the Jones polynomial of an oriented link.

(c) Calculate the Jones polynomial of the following link:



(d) Suppose that the oriented links L_-, L_+, L_0 are skein related and that L_0 is the trefoil knot. Draw a diagram of L_+.