UNIVERSITY OF ABERDEEN **SESSION 2006–07**

DEGREE EXAMINATION
MX4540 Knot Theory
Monday 21 May 2007 (12 noon to 2 pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer THREE questions. All questions carry equal marks.

1. **(a)** Define the Reidemeister moves and state the Reidemeister theorem.

(b) Define the linking number of an oriented link.

(c) Prove that the linking number is a well defined invariant of an oriented link.

(d) Draw a diagram of a link with linking number equal to 4.

(e) Prove that the Hopf link is not equivalent to the Whitehead link.

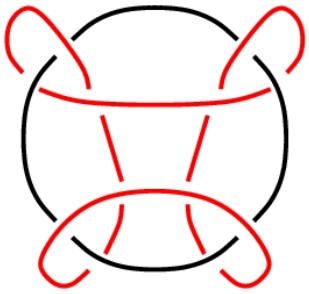
2. **(a)** Define what it means to say that a link L can be coloured `mod n`.

(b) Define what it means to say that a link L is splittable.

(c) Prove that the trefoil and the figure eight knot are not equivalent.

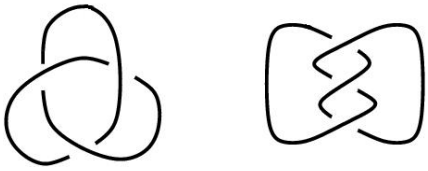
(d) Prove that the Whitehead link is not splittable.

(e) Determine if the following link is splittable.

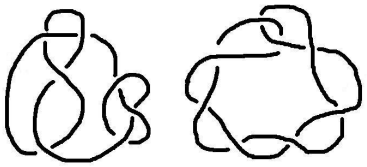


3. Determine which pairs consist of equivalent (oriented) links. Draw relevant pictures if the links are equivalent or give a proof when they are not.

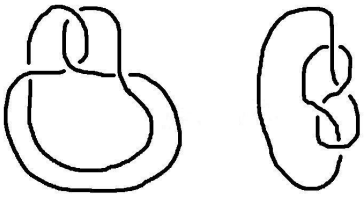
(a)



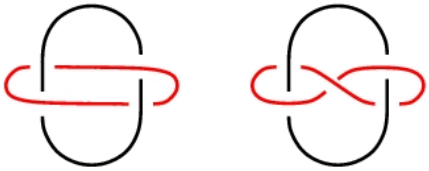
(b)



(c)



(d)



4. Let K be the following knot.

(a) Is K a prime knot? Justify your answer.

(b) Give an example of a colouring of K.

(c) Determine the genus of K.

(d) Determine the Jones polynomial of K.

(e) Is K equivalent to its mirror? Justify your answer.