UNIVERSITY OF ABERDEEN **SESSION 2006–07**

Degree Examination
MX4536 Special Relativity
Wednesday 23 May 2007 (12 noon to 2 pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer THREE questions. The questions are equally weighted.

1. Let I and I' be inertial frames in Special Relativity, in standard configuration, with relative speed v.

(a) Write down the Lorentz Transformations from I to I' and from I' to I.

(b) Events A and B have coordinates $(a, 0, 0, b)$ and $(p, 0, 0, q)$, respectively, in I. It is given that B occurs before A in I. If A and B are simultaneous in I' find an inequality connecting a, b, p, q and c.

(c) State and prove the Lorentz contraction formula as it applies to a rod at rest on the x-axis of the frame I

(d) State and prove the relativistic addition law for velocities, relating the velocity of a particle P in I to its velocity in I'. Particle P has speed u in I and travels along the positive direction of the x-axis in I, whilst particle Q has speed w in I and travels along the positive direction of the y-axis in I. Find their *velocities* in I'. Find also the acute angle between the direction of Q and the negative x'-axis as measured in I'.

2. **(a)** Light is emitted from a source at the spatial origin of an inertial frame I and has frequency ν and wavelength λ in I. It is received at the spatial origin of the frame I' (which is in standard configuration with I and with relative speed v) where it is observed to have frequency ν' and wavelength λ' in I'. Derive the relativistic Doppler formulae relating ν' and ν and relating λ' and λ.

(b) Define the *Lorentz inner product* $<>$ as applied to 4-vectors in Special Relativity and use it to define the terms *spacelike, timelike, null and orthogonal.*

(c) Let X, Y and Z be mutually orthogonal 4-vectors with X and Y spacelike and Z timelike. Let a, b, p and q be any real numbers *with* $p \neq 0$ *and* $q \neq 0$. Show that the 4-vector $aX + bY$ is spacelike. Show also that the 4-vector $pX + qZ$ may be spacelike, timelike or null depending on the choice of p and q.

(d) Show that if U is a timelike 4-vector and L a null 4-vector, then $< U, L > \neq 0$

(e) Let V be a 4-vector with components (V^1, V^2, V^3, V^4) in I and with $V^1 \neq 0$. Show that there exists a frame I', in standard configuration with I, in which the component V'^4 of V is zero, if and only if $V^4/V^1 < 1$.

3. **(a)** In special relativity, a particle P has rest mass m_0 and moves with velocity u in an inertial frame I. Write down expressions for the *inertial mass*, the *momentum* and the *total energy* of P. Write down also expressions for the *momentum* and *total energy* of a photon in I if its frequency in I is ν. State the conservation laws which apply to collisions involving particles and photons.

(b) A particle of mass M_0 is initially at rest in an inertial frame I. It disintegrates completely into two particles P and Q with rest masses m_0' and m_0'', respectively, and which then move with non-zero speeds in I. Show that P and Q move in the same straight line in I. Write down the conservation of energy and momentum equations and deduce that rest mass is not conserved in the disintegration process.

(c) A particle of rest mass m_0 is at rest in I and emits a photon of frequency ν. Show that the speed of recoil u of the particle is given by

$$u = \frac{h\nu c}{m_0 c^2 - h\nu}$$

where h is Planck's constant. If $u = c/2$ show that $\nu = m_0 c^2/3h$

4. Write short essays, including mathematical details where appropriate, on two of the following topics.

(i) The postulates of Special Relativity and their relation to the Michelson-Morley experiment.

(ii) The four dimensional formulation of Special Relativity and the ideas leading to the formula $E = mc^2$.

(iii) The aberration formulae from both the classical and relativistic points of view.