UNIVERSITY OF ABERDEEN **SESSION 2003-04**

DEGREE EXAMINATION

MX3013 Industrial Statistics and Operational Research

Thursday 22 January 2004 (9am to 11am)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer THREE questions. All questions carry equal weight.

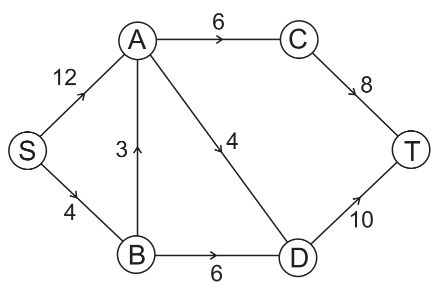
1. **(a)** The following table gives distances between 8 towns, $A - H$.

Find the minimum spanning tree connecting all the towns.

	A	B	C	D	E	F	G	H
A								
B	4							
C	1	2						
D	6	7	7					
E	8	6	2	8				
F	3	1	4	10	5			
G	8	9	7	5	8	9		
H	4	5	2	10	4	4	7	

(b) Explain briefly the type of network problem that can be solved by constructing a minimum spanning tree.

(c) Find the maximum flow from S to T for the directed network below, where the numbers on each arc denote the capacity of that arc. Make sure that you check that your solution is feasible and, by finding the minimum cut, that it is optimal.



(d) Discuss whether the maximum flow can be increased by changing the direction of flow in one of the arcs of the network.

2. **(a)** Suppose that U is a uniform random variable on $[0,1]$ and X is a random variable with cumulative distribution function (c.d.f.) $F(x)$. Show that the random variable $F^{-1}(U)$ has the same distribution as X.

(b) The inversion method is based on the result in (a). Describe in detail how you would use the method to generate a sequence, $X_1, X_2, X_3, \ldots$ of independent random variables, each with c.d.f.

$$F(x) = 1 - e^{-3x^4}, \qquad x > 0$$

given a sequence, $U_1, U_2, U_3, \ldots$ of independent uniform random variables.

(c) Consider a project with seven activities $A - G$. The table below gives, for each activity, the time taken for that activity and the other activities that must precede that activity. Draw the network corresponding to this project.

Activity	**Time Taken**	**Preceding Activities**
A	4	–
B	6	–
C	5	A,B
D	6	B,C
E	6	B
F	3	C
G	2	C,D,E,F

(d) Find the critical path for the network in **(c)**. What is the length of this path?

(e) In the network in **(c), (d)** by how much could you increase the time taken by activity A without increasing the total time taken for the project?

3. **(a)** Explain what is meant by 'M/M/1' for an M/M/1 queue.

(b) Define the *traffic intensity*, ρ, for an M/M/1 queue. For what values of ρ can the queue be in equilibrium?

(c) Consider a single-sever queue for which inter-arrival times and service times have the same distributions as for an M/M/1 queue, but which only has space for a maximum of N customers including the one being served. The probability distribution for the number of customers *in the system* is given on the formula sheet. What is the probability that there are no customers *in the queue*?

(d) Suppose that $N = 2$ in **(c)**, so that there is only one space in which to queue, and $\rho = \frac{2}{3}$.

Show that the probability distribution $p(x)$ for the number of customers in the queue, X, is

x	0	1
$p(x)$	15/19	4/19

(e) For the queueing system in **(d)**, the fixed cost of the server is £200 and there is an additional expected cost of £$133L_q$ where L_q is the expected number of customers in the queue. What is the total expected cost?

(f) For the queueing system in **(d), (e)** there is an option of introducing a faster server. This will reduce the traffic intensity to $\rho = \frac{1}{2}$, but will increase the fixed cost to £210. Calculate whether, in terms of total expected cost, it is worth adopting this option.

4. **(a)** The three updating equations for the multiplicative Holt-Winters procedure are:

$$\hat{\mu}_{t+1} = \alpha_1 \left[\frac{z_{t+1}}{\hat{S}_{t+1-s}}\right] + (1-\alpha_1)(\hat{\mu}_t + \hat{\beta}_t)$$
$$\hat{\beta}_{t+1} = \alpha_2(\hat{\mu}_{t+1} - \hat{\mu}_t) + (1-\alpha_2)\hat{\beta}_t$$
$$\hat{S}_{t+1} = \alpha_3 \left[\frac{z_{t+1}}{\hat{\mu}_{t+1}}\right] + (1-\alpha_3)\hat{S}_{t+1-s}.$$

Define the terms that appear in these equations, and explain the reasoning behind the equations. Write down the forecast equation for this procedure.

(b) The quarterly sales figures of a company have been analysed using the Holt-Winters procedure in **(a)**. Current estimates of μ_t, β_t and S_{t-3} to S_t are

$\hat{\mu}_t$	$\hat{\beta}_t$	$\hat{S}_t$	$\hat{S}_{t-1}$	$\hat{S}_{t-2}$	$\hat{S}_{t-3}$
93	9	0.88	1.01	1.10	1.01

Calculate the forecasts for $(t+1)$ and $(t+2)$.

The actual value for z_{t+1} is 105. Update the forecast for $(t+2)$, using this information and $\alpha_1 = \alpha_2 = \alpha_3 = 0.1$.

(c) Lengths of cable are inspected and the number of defects is recorded. For a sample of 20 lengths the numbers are

1	4	2	0	1	1	0	1	3	3
1	1	0	2	2	0	2	3	8	7

$$\sum x = 42, \qquad \sum x^2 = 178$$

Assuming that the data have a Poisson distribution, calculate an upper control limit, and identify any suspicious observations.

(d) Check whether a Poisson distribution is valid for

(i) the complete data set in **(c)**;

(ii) the data set with any suspicious observations removed.