UNIVERSITY OF ABERDEEN **SESSION 2003–04**

DEGREE EXAMINATION

MA2503 Introduction to Ordinary Differential Equations

Wednesday 26 May 2004 (3pm to 5pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer all FOUR questions. All questions carry equal weight.

1. **(i)** Find the general solution of the differential equation

$$x\frac{dy}{dx} - y = x^2 \sin 2x.$$

(ii) Find the particular solution of the differential equation

$$x^2\frac{dy}{dx} = (y-1)^2, \quad x > 0$$

with initial condition $y = 3$ when $x = 1$.

(iii) Find the general solution of the differential equation

$$\frac{dy}{dx} + y = e^{2x}y^3.$$

2. **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0.$$

Obtain a solution (Particular Integral) of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^{-x}.$$

Write down the general solution of this last equation and also the unique solution satisfying the initial conditions $y = 1$, $\dfrac{dy}{dx} = 0$ when $x = 0$.

(ii) Find the general solution of $\ddot{x} + x = 0$.

Use the method of reduction of order (or otherwise) to obtain the general solution of $\ddot{x} + x = \sin t$.

[Set $x = z(t)\sin t$ and obtain a differential equation for z. Then set $\nu = \dot{z}$ to obtain a first order differential equation for ν. Recall that $\displaystyle\int \operatorname{cosec}^2(t).dt = -\cot(t) = -\frac{\cos t}{\sin t}$.]

3. **(a)** Liquid A is poured into a tank at a rate of 1 litre per minute and liquid B is poured into the tank at a rate of 5 litres per minute. The well-stirred mixture is drawn off at a rate of 2 litres per minute. The tank can hold 800 litres before it overflows. Initially it contains 200 litres of liquid A and no liquid B.

Find an expression for the volume of liquid in the tank after t minutes but before the liquid overflows. How long is it before the liquid overflows?

When the tank overflows, how much liquid B is in the tank?

(b) The position vectors of points on a curve are given by

$$\mathbf{r} = t^2\mathbf{i} - (2t+1)\mathbf{j} + t^3\mathbf{k}.$$

Show that the curve passes through the point with position vector

$$\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Find a unit tangent vector to the curve at this point. Find also the vector equation to the tangent line to the curve at this point.

4. **(i)** A force of 25 Newtons is applied to a particle of mass 5 kilograms in the direction of the vector $4\mathbf{i} - 3\mathbf{k}$. Initially the particle is at O and is moving with speed 10 metres per second in the direction of $\mathbf{j}$. Find the position vector of the particle t seconds later.

(ii) A particle P is projected with speed u metres per second at an angle of elevation α over horizontal ground from a point O on the ground. Find the maximum height of P.

[It should be assumed that the only force acting on the particle is gravity which is producing a constant acceleration g metres per second^2 vertically downwards.]

(iii) A particle of unit mass is dropped vertically downwards from a great height. The forces acting on the particle are gravity as in **(ii)** and also a force due to wind resistance, which is proportional in magnitude to the speed of the particle and acts in the opposite direction to its velocity. Write down the equation of motion of the particle. [You are not asked to solve the equation.]