UNIVERSITY OF ABERDEEN **SESSION 2003–04**

Degree Examination

MA2503 Introduction to Ordinary Differential Equations

Monday 9 August 2004 (12noon to 2pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer all FOUR questions. All questions carry equal weight.

1. **(a)** Find the general solution of the differential equation

$$x\frac{dy}{dx} + y = x^3.$$

(b) Find the general solution of the differential equation $\dfrac{dy}{dx} = 5y$.

Solve the initial value problem $\dfrac{dy}{dx} = 5y$ given that $y = -2$ when $x = 1$.

(c) Solve the initial value problem

$$\frac{dx}{dt} = \frac{x^2 + t^2}{xt},$$

given that $x = 2$ when $t = 1$.

2. **(a)** Find the general solution of the linear second order differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0.$$

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\sin x + 2\cos x.$$

Find also the solution to the last equation which satisfies $y = 0$, $\dfrac{dy}{dx} = 3$ when $x = 0$.

(b) Given that $y = e^x$ satisfies the differential equation

$$(1 + x^2)\frac{d^2y}{dx^2} - (1 + x)^2\frac{dy}{dx} + 2xy = 0,$$

use the method of reduction of order to find the general solution.

3. **(a)** The position vectors of points on a curve are given by

$$\mathbf{r} = -8t\,\mathbf{i} + (2t^3 + 3)\,\mathbf{j} - t^2\,\mathbf{k}, \quad t \in \mathbb{R}.$$

Show that the curve passes through the point $8\mathbf{i} + \mathbf{j} - \mathbf{k}$ and find a unit tangent vector to the curve at this point.

Find the vector and cartesian equation of the tangent line.

(b) Show that the lines

$$\mathbf{r} = (2\mathbf{i} + \mathbf{j}) + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}), \quad \lambda \in \mathbb{R}$$

and

$$\mathbf{r} = (\mathbf{i} + 2\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}), \quad \mu \in \mathbb{R},$$

do not intersect.

Find a vector perpendicular to both lines and the vector equation of a line which intersects both the above lines and is perpendicular to both lines.

4. **(a)** A tank contains a mixture of two liquids A and B. Liquid A is pumped into the tank at a rate of 20 litres per second and the well-stirred mixture is drawn off at a rate of 20 litres per second. Initially there are 1000 litres of liquid A and 4000 litres of liquid B in the tank. How many seconds elapse before there are equal volumes of liquid A and liquid B in the tank?

(b) A force of 4 Newtons acts on a particle of mass 2 in the direction of the vector $\mathbf{i} + \mathbf{k}$. Initially the particle is at a point with position vector $\mathbf{j} + \mathbf{k}$ and has a speed of 2 m/s in the direction of $\mathbf{i} + \mathbf{j}$. Where is the particle 2 seconds after the initial time?

(c) A bullet is fired vertically upwards with speed v at time $t = 0$. Let z be the height of the bullet above the ground. Find a differential equation for z and express z as a function of t.