UNIVERSITY OF ABERDEEN **SESSION 2006–07**

DEGREE EXAMINATION

MA1505 Topics in Mathematics

Wednesday 23 May 2007 (9 am to 11 am)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer one question only from each section. The questions carry an equal number of marks.

Section A

1. Let A and B be sets. Define the sets $A \cup B$, $A \cap B$ and $A \times B$. Let $A = \{a, b, c, d\}$ and $B = \{b, d, e\}$. Write down the sets $A \cup B$ and $A \cap B$. Give an example of an *onto* map $f : A \to B$. Does there exist a *one-to-one* map $f : A \to B$? Justify your answer.

2. Define the terms *one-to-one* and *onto* as applied to a function between two sets.

Let $A = \{a, b, c\}$ and $B = \{d, e, f\}$ be sets and let f be a map from A to B. If f is one-to-one, show that it must be onto.

Give an example of a function $f; \mathbb{R} \to \mathbb{R}$ which is one-to-one but not onto.

Section B

3. **(i)** What is an *octave*, and how many cents are there in an octave?

(ii) What is a *cent*, expressed as a frequency ratio?

(iii) How many cents are there in an interval with frequency ratio 5/3?

4. Two sine waves, with the same peak amplitude A and with frequencies 440Hz and 441Hz, are played together.

(i) Write formulae for these sine waves.

(ii) Rewrite the sum of the two sine waves in part **(i)** as a product of sines and cosines, and interpret the answer in terms of beats.

Section C

5. A committee is to be chosen from a set of 8 women and 5 men and which includes Mr and Mrs Smith. How many ways are there to form the committee if :

(i) The committee has six members, four women and two men ?
(ii) The committee has at least three men and at least twice as many women as men ?
(iii) The committee has four members, at least two of whom are women, and Mr and Mrs Smith cannot *both* be chosen ?

6. How many ways are there to distribute
(i) 6 identical balls a, a, a, a, a, a into 3 identical boxes B, B, B ?
(ii) 6 identical balls a, a, a, a, a, a into 3 distinct boxes A, B, C ?
(iii) 6 distinct balls a, b, c, d, e, f into 3 distinct boxes A, B, C ?

Section D

7. **(a)** Find the shortest distance measured over the surface of the earth between the point with latitude 23° North and longitude 30° East and the point with latitude 39° North and longitude 30° East.

(b) Find the distance *measured along the line of latitude* 50° North between the points of the earth with this latitude and with longitudes 40° East and 45° West, respectively.

[In each case the solution should include a clear diagram. The radius of the earth may be taken as 3960 miles.]

8. **(a)** Describe, briefly, the major differences between the planetary theories of Ptolemy, Copernicus and Kepler.

(b) Given that the semi-major axis of the orbit of the planet Venus about the sun is about 67 million miles, find the approximate time, in days, it takes for Venus to orbit the sun.

[The semi-major axis of the earth's orbit around the sun should be taken to have length 93 million miles.]

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Section E

9. **(a)** Define what it means to say that a group G is cyclic.

(b) Show that the groups $\mathbb{Z}/2 \oplus \mathbb{Z}/3$ and $\mathbb{Z}/6$ are isomorphic.

10. **(a)** Let G and H be groups. Define what it means to say that a map $h : G \to H$ is an isomorphism.

(b) Prove that the groups $\mathbb{Z}/2 \oplus \mathbb{Z}/3$ and $\mathbb{S}_3$ are not isomorphic.

Section F

11. Use the Euclidean algorithm to find the greatest common divisor of 2028 and 4446; show complete working. Find integers s and t such that $2028s + 4446t$ is equal to this greatest common divisor. How can you use your calculator to check your answer, without just repeating the process? (You should do this to make sure you haven't made a mistake)

12. **(i)** Find the multiplicative inverse of 3 modulo 17.

(ii) Use the answer to part (i) to solve the equation $3x \equiv 7 \pmod{17}$.

(iii) Solve the simultaneous equations

$$\begin{aligned} a + b &\equiv 2 \pmod{17} \\ a - 2b &\equiv 3 \pmod{17}. \end{aligned}$$