UNIVERSITY OF ABERDEEN **SESSION 2005–06**

Degree Examination

MA1504 Introductory Mathematics 2

Thursday 1 June 2006 (9 am—11 am)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Show your working clearly in the examination booklet provided and write your final answers in the boxes on this question paper. At the end of the examination, tag together your question paper and your booklet.

Student ID ....

1. Find the centre and radius of the circle

$$x^2 + y^2 - 6x + 10y + 18 = 0.$$

Centre coordinates: $x =$, $y =$
Radius =

2. Given that $y = 2\sin 4x - 5\cos 3x$ find $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$.

$\dfrac{dy}{dx} =$

3. Find the equation of the tangent line to the curve $y = x^2 + 6x - 4$ at the point where $x = -4$.

Tangent Line:

4. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = 4x - 7$. Find the inverse of this function.

$f^{-1}(x) =$

5. If $f(x) = \left(\dfrac{3x+5}{3x-5}\right)^2$ calculate $f'(1)$.

$f'(1) =$

6. Solve the following equation $3^x = (81)^{\frac{1}{5}}$ for x.

$x =$

7. Find the rate of change of $f(x) = \ln(6x+1)$ at the point where $x = 0$.

rate of change $=$

8. Find the magnitude of the vector $\underline{\mathbf{v}} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$.

(Give your answer correct to 2 decimal places.)

$\lvert\underline{\mathbf{v}}\rvert =$

9. Let $A(3, 8, 0)$, $B(1, k, -2)$ and $C(0, 5, -3)$ be three points. Find the value of k which makes these three points collinear.

$k =$

10. Find a unit vector in the direction of the vector $= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(Give each coordinate correct to 2 decimal places.)

Unit vector $=$ $\quad \underline{\mathbf{i}} + \quad \underline{\mathbf{j}} + \quad \underline{\mathbf{k}}$

11. Let $\underline{\mathbf{u}} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$ and $\underline{\mathbf{v}} = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$.

Find (i) $\underline{\mathbf{u}} \cdot \underline{\mathbf{v}}$, and (ii) the angle between $\underline{\mathbf{u}}$ and $\underline{\mathbf{v}}$ (in degrees, correct to 1 decimal place).

(i) $\underline{\mathbf{u}} \cdot \underline{\mathbf{v}}$ (ii) angle $=$

12. Find the indefinite integral $\int \frac{3}{x^3}\, dx$.

Integral:

13. Find the area under the curve $y = \sin 4x$ from $x = 0$ to $x = \frac{\pi}{8}$.

Area =

14. Evaluate $\int_0^3 (x+2)^4\, dx$. (Give your answer correct to 2 decimal places.)

Value =

15. Let $z_1 = -2 - j$ and $z_2 = 5 - j$. Find $\frac{z_1}{z_2}$ in the form $a + bj$ where a and b are fractions in lowest terms.

$\frac{z_1}{z_2} =$

16. Express $-4 + 5j$ in polar form. Give the angle in degrees, between 0 and 360. Give both answers correct to 2 decimal places.

Polar form = ______$\measuredangle$______°

17. Determine the quadratic equation in z which has $4 - 5j$ as one of its roots.

Equation:

18. Evaluate the determinant of the matrix $A = \begin{bmatrix} -2 & 10 \\ -5 & 3 \end{bmatrix}$.

Determinant =

19. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 5 \\ 2 & -1 \end{bmatrix}$.

$A^{-1} =$

20. Let $A = \begin{bmatrix} -2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & -2 \\ 1 & 4 \end{bmatrix}$. Find the matrix product AB.

$AB =$

21. If $f(x) = kx^3 + 5x + 1$ and $f'(1) = 2$ find the value of k.

$k =$

22. Find the value of x for which the following function has a local maximum:

$$y = 2x^3 - 3x^2 - 120x - 6.$$

$x =$

23. A particle is moving in a straight line. Its distance s from a fixed point on the line is given by

$$s = -t^2 + 4t + 2.$$

At time $t = 2$ find: (i) the velocity of the particle and (ii) the acceleration of the particle.

velocity $v =$
acceleration $a =$

24. Find the particular solution of the differential equation

$$\frac{dy}{dx} = 3x^5$$

given that $y = \frac{3}{4}$ when $x = 1$. Give coefficients and constants as fractions.

$y =$