UNIVERSITY OF ABERDEEN **SESSION 2004–2005**

Degree Examination

MA1504 Introductory Mathematics 2

Thursday 2 June 2005 (12 noon—2 pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Show your working clearly in the examination booklet provided and write your final answers in the boxes on this question paper. At the end of the examination, tag together your question paper and your booklet.

Student ID ....

1. Find the centre and radius of the circle

$$x^2 + y^2 + 10x - 6y - 66 = 0.$$

(2 marks)

Centre coordinates: $x =$, $y =$
Radius $=$

2. Given that $0 \leq \theta \leq \frac{\pi}{2}$ and $\cos\theta = \frac{5}{13}$, find the exact value of $\sin 2\theta$ as a fraction in lowest terms.

(2 marks)

$\sin 2\theta =$

3. Find $f'(x)$ where $f(x) = \sqrt{x^3 + 2x^2 - 4x + 2}$.

(2 marks)

$f'(x) =$

4. Given that $y = 4\sin 2x - 2\cos 3x$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

(2 marks)

$\frac{dy}{dx} =$

5. Let $f : R \to R$ be defined by $f(x) = 9x - 7$.

Find the inverse of this function.

(2 marks)

$f^{-1}(x) =$

6. Given that $f(x) = x^3 \sin(2x)$, find the slope of the tangent to the curve $y = f(x)$ at the point where $x = 3$ radians. Give the answer correct to 2 d.p.

(2 marks)

tangent slope $=$

7. Given that $y = \dfrac{x^3}{x^5 + 2}$, find the equation of the tangent to the curve at $x = 1$.

(4 marks)

$y =$

8. Find the rate of change of $f(x) = \ln(5x + 3)$ at the point where $x = 0$.

(2 marks)

rate $=$

9. Find $f'(1)$ where $f(x) = x^6 e^{4x}$. Give the answer correct to 2 d.p.

(2 marks)

$f'(1) =$

10. A is the point $(7, 1)$ and B is the point $(-6, 4)$. Find (i) the mid-point of AB, (ii) the slope of AB, (iii) the equation of the perpendicular bisector of AB (in the form $y = mx + c$ where m and c are fractions).

(4 marks)

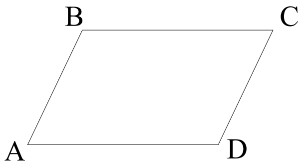
(i) $x =$, $y =$
(ii) slope of AB $=$
(iii)

11. Given $\mathbf{u} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 6 \\ 5 \end{bmatrix}$, find $2\mathbf{u} - 4\mathbf{v}$.

(2 marks)

$2\mathbf{u} - 4\mathbf{v} = \begin{bmatrix} \\ \\ \end{bmatrix}$

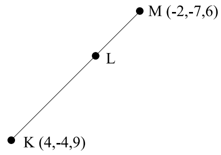
12. Given $A(5,5,0)$, $B(-6,-1,0)$ and $C(-4,-6,4)$ as vertices of a parallelogram, find the coordinates of the point $D(d_1, d_2, d_3)$.



(2 marks)

$d_1 =$, $d_2 =$, $d_3 =$

13. The point L divides KM in the ratio $2:1$ as shown in the diagram. Find the coordinates of L.



(2 marks)

$x =$, $y =$, $z =$

14. Let $\mathbf{u} = \begin{bmatrix} 3 \\ -9 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 7 \\ -7 \\ 7 \end{bmatrix}$.

Find (i) $\mathbf{u}.\mathbf{v}$, (ii) the angle between $\mathbf{u}$ and $\mathbf{v}$ (in degrees, to 2 d.p.).

(3 marks)

(i) $\mathbf{u} \cdot \mathbf{v} =$
(ii) angle $=$

15. Find the value of b such that $\mathbf{u} = b\mathbf{i} + (1-b)\mathbf{j} - 2\mathbf{k}$ is orthogonal to $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Express b as a fraction in lowest terms.

(2 marks)

$b =$

16. Find the indefinite integral $\int \frac{2}{x^3}\, dx$.

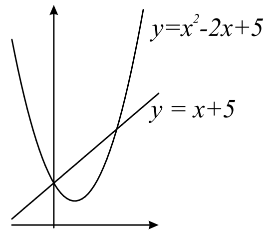
(2 marks)

17. Evaluate $\int_0^{\pi/3} (-2\sin 4x - \cos 2x)\, dx$ correct to 2 d.p.

(2 marks)

Value $=$

18. The diagram shows the line $y = x + 5$ and the curve $y = x^2 - 2x + 5$.
Construct the integral which represents the area between the line and the curve. Do not carry out the integration.



(2 marks)

$$\int_{x=}^{x=} [\qquad\qquad\qquad] \, dx$$

19. Let $z_1 = 2 - 2j$ and $z_2 = 5 - j$. Find $\frac{z_1}{z_2}$ in the form $a + bj$ where a and b are fractions in lowest terms.

(2 marks)

$\frac{z_1}{z_2} =$

20. Express $4 - 5j$ in polar form. Give the angle in degrees (between 0 and 360). Give both parts of the answer correct to 2 d.p.

(2 marks)

Polar Form = $\square \angle \square$ °

21. Let $B = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 2 & 5 \\ -1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}$. Give the size of CD and give the elements of BC.

(2 marks)

CD is $\square$ by $\square$;

BC = $\begin{bmatrix} \quad \end{bmatrix}$

22. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 2 \\ -3 & 2 \end{bmatrix}$. Write the elements as fractions.

(2 marks)

$A^{-1} = \begin{bmatrix} \quad \end{bmatrix}$

23. Find the value of x for which the following function has a local maximum.

$$y = 2x^3 + 3x^2 - 12x - 6$$

(2 marks)

$x =$

24. A particle is moving on a straight line. Its distance s from a fixed point on the line is given by

$$s = t^2 - 2t + 4.$$

At time $t = 4$, find (a) the velocity of the particle, (b) the acceleration of the particle.

(2 marks)

velocity v =
acceleration a =

25. Find the particular solution of the differential equation

$$\frac{dy}{dx} = 2x^4,$$

given that $y = \frac{5}{2}$ when $x = 1$. Give coefficients and constants as fractions.

(2 marks)

y =