UNIVERSITY OF ABERDEEN **SESSION 2003–2004**

Degree Examination

MA1504 Introductory Mathematics 2

Tuesday 25 May 2004 (12noon to 2pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer SIX questions. All questions carry equal weight.

1. **(a)** Find the derivatives of the following functions:

$$\text{(i)}\ \frac{1}{4}x^4 + \frac{4}{x}, \quad \text{(ii)}\ \cos^3 x, \quad \text{(iii)}\ (x^2+3x+1)^{10}, \quad \text{(iv)}\ \frac{t}{t^2+1}.$$

(b) Find the constants a and b such that the function $f(x) = x(a\cos x + b\sin x)$ satisfies the equation:

$$f''(x) + f(x) = \sin x.$$

2. **(a)** Find the stationary points of the function $f(x) = x^4 - 2x^2$, and determine their nature. Also determine the intervals on which the function is increasing and those on which it is decreasing. Roughly sketch the graph $y = f(x)$.

(b) Let $g(x) = \mathrm{e}^{-x^2}$. Show that g has one stationary point and determine its nature.

3. **(a)** Let $f(x) = \ln x + x - 2$. Show that the equation $f(x) = 0$ has a solution in the interval $[1, 2]$. Use Newton's method to obtain an approximation to this solution starting with $x_1 = 1$ and using two iterations. [Recall: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$]

(b) The velocity of a particle moving in a straight line at time t seconds is $v(t)$ m/s where $v(t)$ is given by the formula

$$v(t) = 24t - 3t^2.$$

(i) Find the acceleration of the particle when $t = 2$ seconds.

(ii) Find the time at which the velocity is greatest and find the maximum velocity.

4. **(a)** Solve the following quadratic equation, expressing the solutions in the form $x+y\mathrm{i}$ (where x and y are real):

$$z^2 - 14z + 53 = 0\,.$$

(b) Let $z = 5 + 2\mathrm{i}$, $w = 21 - 20\mathrm{i}$.

(i) Determine the complex conjugate, $\overline{w}$, and modulus, $|w|$, of w.

(ii) Express zw and z/w in the form $x + y\mathrm{i}$.

(c) Express the complex number $1 - \mathrm{i}\sqrt{3}$ in polar form $r(\cos(\theta) + \mathrm{i}\sin(\theta))$ (where $r > 0$).

5. **(a)** Let $A = \begin{bmatrix} 7 & 3 \\ 11 & 5 \end{bmatrix}$.

(i) Show that $A^2 - 12A + 2I = 0$.

(ii) Determine $\det(A)$ and A^{-1}.

(iii) Use A^{-1} to solve the system of equations

$$\begin{array}{rcrcl} 7x & + & 3y & = & 1 \\ 11x & + & 5y & = & 3. \end{array}$$

(b) Find those values of the constant λ for which the matrix

$$\begin{bmatrix} 6-\lambda & 2 \\ 1 & 5-\lambda \end{bmatrix}$$

does not have an inverse.

6. **(a)** Let $\mathbf{a} = 3\mathbf{i} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Determine $|\mathbf{a}|$, $|\mathbf{b}|$ and $\mathbf{a} \cdot \mathbf{b}$. Hence find the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$.

(b) In a triangle ABC, let $\mathbf{u} = \overrightarrow{AB}$, $\mathbf{v} = \overrightarrow{BC}$ and $\mathbf{w} = \overrightarrow{AC}$. Let P and Q be the midpoints of the sides AB and BC, respectively.

(i) Express $\mathbf{w}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(ii) Express $\overrightarrow{AP}$, $\overrightarrow{BQ}$ and $\overrightarrow{PQ}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(iii) Deduce that the vector $\overrightarrow{PQ}$ is parallel to $\mathbf{w}$, and determine the ratio $|\overrightarrow{PQ}|/|\mathbf{w}|$.

7. **(a)** Find

$$\int (\mathrm{e}^{-x} + \mathrm{e}^{-2x})\, dx.$$

(b) Use partial fractions to find

$$\int \frac{1}{x(x+1)}\, dx.$$

(c) Find the area enclosed between the curve $y = \frac{1}{2}x^2$ and the line $y = x$. [It will be helpful to draw a sketch.]

8. **(a)** Evaluate

$$\int_0^3 (2x+1)^2\, dx.$$

(b) Use the substitution $u = \frac{1}{2}x^2$ to evaluate

$$\int_0^4 x\,\mathrm{e}^{-\frac{1}{2}x^2}\, dx.$$

(c) Use integration by parts to evaluate

$$\int_0^1 x\,\mathrm{e}^{-x}\, dx.$$