UNIVERSITY OF ABERDEEN **SESSION 2003–2004**

DEGREE EXAMINATION

MA1504 Introductory Mathematics 2

Tuesday 10 August 2004 (12noon to 2pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer SIX questions. All questions carry equal weight.

1. Find the derivatives of the following functions:

$$\text{(i) } x\cos x, \quad \text{(ii) } \ln(1+t^2), \quad \text{(iii) } (1+\sqrt{t})^2,$$

$$\text{(iv) } \frac{1-x}{1+2x}, \quad \text{(v) } \mathrm{e}^{2\theta}+\sin 2\theta, \quad \text{(vi) } x+\frac{1}{x^2-3}.$$

2. **(a)** Find the three stationary points of the function $f(x) = x^3(x-1)^2$, and determine their nature. Also determine the intervals on which the function is increasing and those on which it is decreasing. Roughly sketch the graph $y = f(x)$.

(b) Let $g(x) = a\mathrm{e}^{-x} + b\mathrm{e}^{3x}$, where a, b are any constants. Show that g satisfies the equation

$$g''(x) - 2g'(x) - 3g(x) = 0.$$

3. The acceleration of a particle moving in a straight line at time t seconds is $a(t)$ where $a(t) = 12 - 6t$. The initial velocity of the particle is -9 m/s and it is initially at a point 2m to the right of a fixed point O on the line.

(i) Find the velocity of the particle at time t.

(ii) Find the position of the particle relative to O at time t.

(iii) Find the times at which the particle comes instantaneously to rest.

(iv) Find its position at these times.

(v) Show that the particle passes through O after 2 seconds and find its velocity at that time.

(vi) Describe the motion of the particle.

4. **(a)** Solve the following quadratic equation, expressing the solutions in the form $x+y\mathrm{i}$ (where x and y are real):

$$z^2 + 6z + 34 = 0\,.$$

(b) Let $z = 2 - \mathrm{i}$, $w = 3 + 4\mathrm{i}$.

(i) Determine the complex conjugate, $\overline{w}$, and modulus, $|w|$, of w.

(ii) Express zw and z/w in the form $x + y\mathrm{i}$.

(c) Express the complex number $-3 + 3\mathrm{i}$ in polar form $r(\cos(\theta) + \mathrm{i}\sin(\theta))$ (where $r > 0$).

5. **(a)** Let $A = \begin{bmatrix} 4 & 7 \\ 3 & 6 \end{bmatrix}$.

(i) Show that $A^2 - 10A + 3I = 0$.

(ii) Determine $\det(A)$ and A^{-1}.

(iii) Use A^{-1} to solve the system of equations

$$\begin{aligned} 4x &+ 7y &= 19 \\ 3x &+ 6y &= 12. \end{aligned}$$

(b) Find those values of the constant λ for which the matrix

$$\begin{bmatrix} 10-\lambda & -4 \\ 5 & 1-\lambda \end{bmatrix}$$

does not have an inverse.

6. **(a)** Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. Determine $|\mathbf{a}|$, $|\mathbf{b}|$ and $\mathbf{a}\cdot\mathbf{b}$. Hence calculate the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$.

(b) In a triangle ABC, let $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AC}$. Let D be the midpoint of the side BC.

(i) Express $\overrightarrow{BC}$, $\overrightarrow{BD}$ and $\overrightarrow{AD}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(ii) Suppose that $|\mathbf{u}| = |\mathbf{v}|$. Show (using vector methods) that $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$.

7. **(a)** Find A, B such that

$$\frac{x+1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}.$$

Hence find

$$\int \frac{x+1}{x(x-2)}\,dx.$$

(b) Find the area enclosed between the curve $y = 1 + x^2$ and the line $y = 1 - x$. [It will be helpful to draw a sketch.]

8. **(a)** Evaluate the following definite integrals:

$$\int_0^1 (x^2 - 1)^2\, dx, \quad \int_0^{\pi/2} (\cos 3x + \mathrm{e}^{-x})\, dx, \quad \int_1^2 \frac{1}{x}\, dx.$$

(b) Use the substitution $u = 1 + x^3$ to evaluate

$$\int_0^2 x^2\, (1 + x^3)^{1/2}\, dx.$$