UNIVERSITY OF ABERDEEN **SESSION 2003–2004**

DEGREE EXAMINATION

MA1004 Introductory Mathematics 1

Tuesday 27 January 2004 (9am to 11am)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer SIX questions. All questions carry equal weight.

1. **(a)** Simplify:

$$\text{(i)} \quad \frac{x^3\,y^4}{x^2\,y^{-1}}; \qquad \text{(ii)} \quad \frac{\left(27a^9\,b^6\,c^{\frac{3}{2}}\right)^{\frac{1}{3}}}{a^3\,b\,c^{\frac{1}{2}}}.$$

(b) Expand $(2y+z)^2$.

(c) Express a in terms of x if $\dfrac{3}{x} = \dfrac{a+2}{a-1}$.

(d) Simplify $\dfrac{a^2-b^2}{a^3+a^2b}$.

2. Let l_1 be the line through the points $(-3,\,3)$ and $(-1,\,2)$. Find an equation for l_1.

Let l_2 be the line through the point $(0,\,-1)$ with slope 2. Find an equation for l_2.

Are l_1 and l_2 perpendicular? Justify your answer.

Find the point of intersection P of the lines l_1 and l_2.

Find the point Q at which l_1 meets the x-axis.

Determine the distance from P to Q.

3. **(a)** By "completing the square", find the minimum value of x^2+6x-5 and the value of x at which the minimum is achieved. What is the maximum value of $5-6x-x^2$?

(b) Let $p(x) = x^3 - 2x^2 - 5x + 6$. Show that $p(1) = 0$. Factorise the polynomial $p(x)$ completely and hence find all the solutions of the equation $p(x) = 0$.

4. **(a)** Convert 240° to radians, expressing your answer as a multiple of π.

(b) Find the length of an arc of a circle of radius 20 cm which subtends an angle 60° at the centre of the circle.

(c) Express $3\sin x^\circ - 4\cos x^\circ$ in the form $R\cos(x-\alpha)^\circ$ where $R>0$ and $0 \leqslant \alpha \leqslant 360$.

Hence sketch the graph $y = 3\sin x^\circ - 4\cos x^\circ \quad (0 \leqslant x \leqslant 360)$.

5. **(a)** Express $2\log(A) - 3\log(B) + 4\log(C)$ in terms of a single logarithm.

(b) Solve the equation $4^x = 7$.

(c) A building society pays interest on its accounts. The amount of money m in a particular account is given by

$$m = ce^{kt} \text{ pounds},$$

where t is the time in years and c and k are constants. The account contains 100 pounds when $t = 0$ and this increases to 150 pounds after 10 years. Find the values of c and k and determine the amount in the account after 20 years to the nearest pound.

6. **(a)** Find $f'(x)$ in each of the cases:

(i) $f(x) = (x+1)(x-2)$;

(ii) $f(x) = \dfrac{3}{x} - \dfrac{1}{x^2}$;

(iii) $f(x) = 3\cos(2x) - 2\sin(3x)$.

(b) Find an equation of the tangent to the graph $y = x^3 - 3x + 1$ at the point on the graph where $x = 2$.

(c) Find an equation of the tangent to the graph $y = \sin(x) + \cos(x) + 3$ at the point on the graph where $x = \pi$.

7. **(a)** Find $f'(x)$ in both of the cases:

(i) $f(x) = \sqrt{x}(x+1)$;

(ii) $f(x) = 3\sin^2(x) + (3x + \cos(x))^4$;

(b) Let $f(x) = 6x - 2x^3$. Find the stationary points of f and determine the nature of each stationary point. Also find the intervals on which f is increasing or decreasing.

8. **(a)** Find:

$$\text{(i)} \quad \int (x^3 + x + 7)\,dx; \qquad \text{(ii)} \quad \int (\cos(2x) + \sin(7x))\,dx.$$

(b) Evaluate:

$$\int_0^1 \left(x^2 - \sin(\pi x)\right)\,dx.$$

(c) Find where the line $y = 2x$ and the graph $y = 2x(3 - x)$ intersect.

Find the (finite) area enclosed by the line and graph.