UNIVERSITY OF ABERDEEN **SESSION 2003–2004**

Degree Examination

MA1004 Introductory Mathematics 1

Tuesday 17 August 2004 (3pm to 5pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer SIX questions. All questions carry equal weight.

1. **(a)** Simplify: (i) $\dfrac{\left(\frac{x}{y}\right)^3}{\left(\frac{1}{y}\right)^4}$; (ii) $\dfrac{\left(8\,a^3\,b^9\,c^7\right)^{\frac{1}{3}}}{2\,a\,b^2\,c^{\frac{1}{4}}}$.

(b) Expand $(2s+3t)^2$.

(c) Express x in terms of y if $\dfrac{2}{x+3} = \dfrac{y}{x-1}$.

(d) Simplify $\dfrac{a^2+2ab+b^2}{a^2-b^2}$.

2. Let L_1 be the line through the points $(-2,\,2)$ and $(13,\,-3)$. Find an equation for L_1.

Let L_2 be the line through the point $(2,\,4)$ which is perpendicular to L_1. Find an equation for L_2.

Find the point of intersection P of the lines L_1 and L_2.

Find the point Q at which L_1 meets the y-axis.

Determine the distance from P to Q.

3. **(a)** By "completing the square", find the minimum value of $2x^2+12x+13$ and the value of x at which the minimum is achieved.

(b) Let $p(x) = x^3+4x^2+x-6$. Show that $p(1)=0$. Find all the solutions of the equation $p(x)=0$ and factorise the polynomial $p(x)$ completely.

(c) Let $f(x) = x^3+2x-1$. Show that the equation $f(x)=0$ has a solution lying between 0 and 1. Use the bisection method to find an approximation to this solution with error less than 0.3.

4. **(a)** The length of an arc of a circle of radius 40 cm is 45 cm. Find the angle subtended at the centre of the circle by the arc in degrees.

(b) If $\cos\theta = \frac{1}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$ and $0 \leqslant \theta < 2\pi$, find the exact value of θ in radians.

(c) Express $3\cos x^\circ + 2\sin x^\circ$ in the form $R\cos(x-\alpha)^\circ$ where $R>0$ and $0 \leqslant \alpha \leqslant 360$.

Hence sketch the graph $y = 3\cos x^\circ + 2\sin x^\circ \quad (0 \leqslant x \leqslant 360)$.

5. **(a)** Without using a calculator, evaluate

$$\log_{10}(\frac{3}{2}) + \log_{10}(\frac{2}{5}) + \log_{10}(\frac{5}{3}).$$

Show your calculations (zero marks will be awarded if you just give the answer).

(b) Solve the equation $3^x = 40$, giving an answer accurate to three decimal places.

(c) The population P of an Eastern European country is given by

$$P = ce^{\alpha t} \text{ millions,}$$

where t is the time in years and c and α are constants. The population is 17.58 million when $t = 0$ and 21.23 million when $t = 5$. Find the values of c and α and determine the population when $t = 10$.

6. **(a)** Find $f'(x)$ in each of the cases:

(i) $f(x) = x(x + 2)$;

(ii) $f(x) = \dfrac{1}{x} + \dfrac{3}{x^3}$;

(iii) $f(x) = 3\cos(5x)$.

(b) Find an equation of the tangent to the graph $y = x^3 + 2x^2 - x + 1$ at the point on the graph where $x = 2$.

(c) Find the coordinates of the point on the graph $y = x^2 + 2x - 3$ at which the tangent is parallel to the line $y = 3x + 1$.

7. **(a)** Find $f'(x)$ in both of the cases:

(i) $f(x) = \sqrt{x + 1}$;

(ii) $f(x) = \cos^2(x) + (x + \sin(x))^3$;

(b) Let $f(x) = x^3 - 3x$. Find the stationary points of f and determine the nature of each stationary point. Also find the intervals on which f is increasing or decreasing.

8. **(a)** Find:

(i) $\displaystyle\int (x^3 + x^2)\, dx$; (ii) $\displaystyle\int (\cos(x) + \sin(3x) + 1)\, dx$.

(b) Evaluate:

$$\int_0^1 \left(x^3 - \cos(\pi x)\right) dx.$$

(c) Find where the line $y = x$ and the graph $y = x(3 - x)$ intersect.

Find the (finite) area enclosed by the line and graph.