UNIVERSITY OF ABERDEEN **SESSION 2004–05**

Degree Examination

MA1002 Calculus

Tuesday 25 January 2005 (3pm to 5pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Attempt ALL FIVE of the questions in SECTION A and TWO of the questions in SECTION B. Each question in section A is worth 12 marks and each question in section B is worth 20 marks.

SECTION A

1. Differentiate the following functions

$$f(x) = 2 - \cos(3x-1) + 4x^{-3/2}, \qquad g(x) = \frac{x^2+3}{x^2+x+1},$$
$$h(x) = x^3 e^{-x}, \qquad k(x) = (1+\ln(2x))^{-1}.$$

2. **(a)** Find the indefinite integrals

$$\int (2 - \cos(3x-1) + 4x^{-3/2})\,dx, \qquad \int \left(e^{-x} - \frac{1}{\sqrt{1-x^2}}\right) dx.$$

(b) Evaluate the definite integrals

$$\int_0^1 \frac{x}{\sqrt{1+2x^2}}\,dx, \qquad \int_0^1 te^{2t}\,dt.$$

3. **(a)** Sketch the graphs $y = x^2+1$ and $y = 9-x^2$. Find the points where the graphs meet and then find the area of the region lying between the two graphs.

(b) What is the volume of the solid of revolution obtained by taking the part of the graph of $y = \sqrt{x}$ between $x=0$ and $x = a > 0$ and rotating it about the x-axis?

4. **(a)** Find the critical points of the function $f(x) = x^4 - 2x^3 + 1$. Find the maximum and minimum values of $f(x)$ in the interval $-1 \leqslant x \leqslant 2$.

(b) Where does $\dfrac{x^3}{\ln(x)}$ achieve its minimum value for $x > 1$?

5. By sketching the graphs $y = 1 + \dfrac{1}{x}$ and $y = x^2$ for positive values of x, explain why the equation $1 + \dfrac{1}{x} = x^2$ only has one positive solution and use the Newton-Raphson method to find an approximation to this solution, stating your result accurate to 3 decimal places.

SECTION B

6. **(a)** Suppose that A and B are constants and that $A\sin x + B\cos x = 0$ for all values of x. By considering particular values of x, or otherwise, prove that $A = B = 0$.

(b) Let $y = e^{px}(4\sin 2x - 5\cos 2x)$, where p is a constant. Show that

$$y'' = e^{px}\left\{(4p^2 + 20p - 16)\sin 2x - (5p^2 - 16p - 20)\cos 2x\right\}.$$

For what value of p does y satisfy the differential equation $y'' - 2y' + 5y = 0$?

7. **(a)** Show that

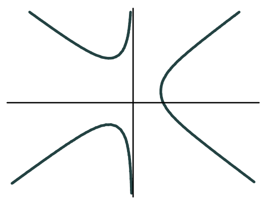
$$\int_0^1 \frac{2 - 3x}{(x+1)(2-x)(3-x)}\,dx = \frac{7}{4}\ln 3 - \frac{8}{3}\ln 2.$$

What comment would you make about the integral $\displaystyle\int_0^1 \frac{dx}{2x - 1}$?

(b) Find the solution of the equation $x\dfrac{dy}{dx} = 1 + y$ which satisfies y=0 when x=1.

8. Find the equation of the tangent to the curve given by $x^3 + yx - xy^2 = 1$ at the point $(1, 0)$.

The diagram shows a sketch of this curve (together with the axes). Noting that the equation for the curve is a quadratic in y, solve for y and explain why y is approximately $\frac{1}{2} \pm x$ for large values of x (positive or negative).



9. **(a)** Define the function $\arctan(x)$ and prove that its derivative is $1/(1 + x^2)$. You may assume that the derivative of $\tan(x)$ is $\sec^2(x)$.

(b) Find the first three terms of the Maclaurin series for $f(\lambda) = e^{\lambda\cos\theta}$, where θ is fixed. Hence show that for small values of λ, to this level of approximation,

$$\frac{1}{2\pi}\int_0^{2\pi} e^{\lambda\cos\theta}\,d\theta \approx 1 + \frac{1}{4}\lambda^2.$$