UNIVERSITY OF ABERDEEN **SESSION 2003–04**

Degree Examination

MA1002 Calculus

Wednesday 28 January 2004 (12noon - 2pm)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Attempt ALL FIVE of the questions in SECTION A and TWO of the questions in SECTION B. Each question in section A is worth 12 marks and each question in section B is worth 20 marks.

SECTION A

1. Differentiate the following functions

$$f(x) = 4x^2 - 3x^{-5/3}, \qquad g(x) = \frac{3x+1}{2x^2+x}, \qquad h(x) = e^x \sin(x^2), \qquad k(x) = \sin(x + \cos(x)).$$

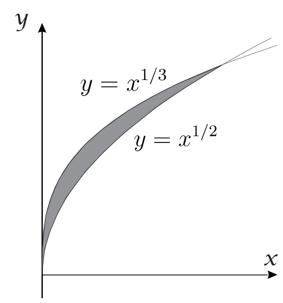
2. **(a)** Find the indefinite integrals

$$\int (x^4 + \cos 2x + x^{-1/3})\, dx, \qquad \int \left(e^{2x} + \frac{1}{1+x^2}\right) dx.$$

(b) Evaluate the definite integrals

$$\int_0^1 \frac{x^3}{1+x^4}\, dx, \qquad \int_0^{\pi/2} t \sin t\, dt.$$

3. **(a)** Find the points in the first quadrant where the curves $y = x^{1/2}$ and $y = x^{1/3}$ meet and then find the area of the shaded region lying between the curves.



(b) Evaluate the definite integral $\displaystyle\int_0^{\pi/2} (1 + \cos\theta)^2 \sin\theta\, d\theta$.

4. **(a)** Find the critical points of the function $f(x) = x^3 - 12x + 1$. Determine which of these points are local maxima and which are local minima. Find the maximum and minimum values of $f(x)$ in the interval $[0, 2]$.

(b) Give the general solution of the differential equation $\dfrac{dy}{dx} = ky$, where k is a constant. If $y = 1$ when $x = 0$ and $y = 2$ when $x = 1$, show that $y = 4$ when $x = 2$.

5. Use the Newton-Raphson method to find the solution to the equation $x^3 = x^2 + 1$ that is between $x = 1.4$ and $x = 1.6$. Start from $x = 1.5$ and state your result accurate to 3 decimal places.

SECTION B

6. Let $y = e^{-x}(\sin(ax) - 3\cos(ax))$, where a is a constant. Show that

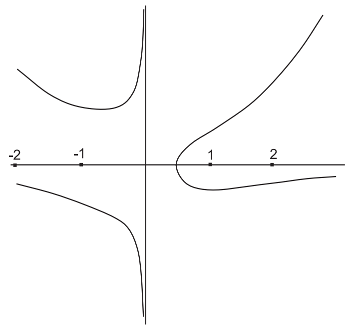
$$y'' = e^{-x}\left((3a^2 - 2a - 3)\cos(ax) - (a^2 + 6a - 1)\sin(ax)\right).$$

For what values of a does y satisfy the differential equation $y'' + 2y' + 5y = 0$?

7. **(a)** Evaluate $\displaystyle\int_0^1 \frac{x + 2\,dx}{(2x+1)(x+3)}$.

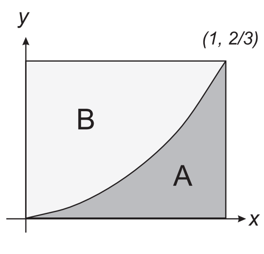
(b) Find the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x\cos^2 y}{x^2 + 1}$ which satisfies $y = 0$ when $x = 2$.

8. Consider the curve given by the equation $2x + x^3y - 2xy^2 = 1$. Part of the curve is shown in the accompanying picture.



Find the equation of the tangent to this curve at the point $(1, 1)$ on the curve.
Confirm one detail of the diagram by proving that there must be points of the curve corresponding to any value of x in the ranges $x < 0$ and $x \geqslant 1/2$.
Explain why the section of the curve that lies in the first quadrant is approximately given by $y = \frac{1}{2}x^2$ when x and y are large.

9. The sketch is of the graph of $y = \frac{2}{3}x^{3/2}$ for $0 \leqslant x \leqslant 1$.



Calculate

(i) The volume of the solid generated when the area A is rotated through 2π radians about the x-axis.

(ii) The volume of the solid generated when the area B is rotated through 2π radians about the y-axis.

(iii) The length of the curve from $(0, 0)$ to $(1, 2/3)$.