

DEGREE EXAMINATION

MA1002 Calculus

Wednesday, 17 January 2007

(9 am to 11 am)

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*Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.*

*Marks may be deducted for answers that do not show clearly how the solution is reached.*

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Attempt All FIVE of the questions in SECTION A and TWO of the questions in SECTION B. Each question in Section A is worth 12 marks and each question in Section B is worth 20 marks.

**SECTION A**

1. Differentiate the following functions

$$f(x) = 3x^5 + e^{6x} + 2x^{1/3}, \quad g(x) = \frac{3 + x^2}{1 + x}, \quad h(x) = x^3 \cos x$$

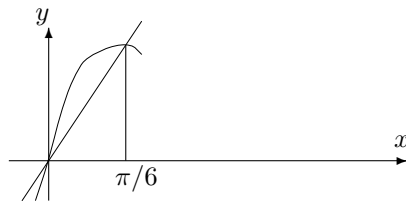
2. (a) Find the indefinite integrals

$$\int (x^2 - 2x^{-3} + 5e^{4x}) dx, \quad \int (4 \cos 3x + \frac{1}{3x-2}) dx$$

- (b) Evaluate the definite integral

$$\int_1^e \frac{1}{x(1 + \ln x)^2} dx$$

3. (a) Determine the area between the curve  $y = \sin 3x$  and the line  $y = \frac{6x}{\pi}$  in the first quadrant.



- (b) Use integration by parts to show that

$$\int_0^{\pi/2} t^2 \sin t dt = \pi - 2$$

4. (a) Find the critical points of the function  $f(x) = 2x^3 + 5x^2 - 4x + 1$  and hence find the maximum and minimum values of this function when  $-2 \leq x \leq 1$ .

(b) Show that the function  $g(x) = \cos(x^2)$  has a critical point at  $x = 0$ . Is it a maximum, a minimum or neither?

5. (a) Find the equation of the line tangent to the graph of  $f(x) = |1 - x^2|$  at the point where  $x = 3$ .

(b) Find the equation of the line having slope  $\frac{1}{10}$  that is tangent to the curve

$$y = \sqrt{x+3}.$$

### SECTION B

6. (a) Show that  $\int_3^4 \frac{3x}{(2x+1)(x-1)} dx = \ln \frac{9}{2\sqrt{7}}$

(b) The region bounded by the graph of  $y = x^2 - \frac{1}{\sqrt{x}}$  and the  $x$ -axis between  $x = 1$  and  $x = 2$  is rotated about the  $x$ -axis. Calculate the volume of the solid of revolution so obtained.

7. (a) Find the area of the triangle formed by the  $x$ -axis, the  $y$ -axis and the tangent at the point  $(1, -1)$  to the curve given implicitly by the equation

$$x^3 + 3xy^2 + y^3 - 3 = 0.$$

(b) Find the values of the constants  $a$  and  $b$  such that  $f$  is a continuous function

$$f(x) = \begin{cases} x^3 - 2x^2 + 1, & \text{if } x \leq 2 \\ \frac{5}{2}x + a, & \text{if } 2 < x \leq 4 \\ bx^2 + 3x, & \text{if } x > 4 \end{cases}$$

8. Calculate the first two derivatives of the function

$$h(x) = \frac{e^x}{x-1}$$

and use this information to sketch the graph of the function  $h(x)$ .

9. A tank is in the form of a cylinder with a circular base and top. The volume of the tank is  $V$ .

Find the values of the radius  $r$  and height  $h$  that minimise the overall area of the tank (curved sides and flat top and bottom).

(*Indication* : The area of the curved surface of such a cylinder is  $2\pi rh$ , the volume is  $\pi r^2 h$ .)