UNIVERSITY OF ABERDEEN **SESSION 2003–04**

DEGREE EXAMINATION

MA1002 Calculus

Wednesday 18 August 2004 (9am to 11am)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Attempt ALL FIVE of the questions in SECTION A and TWO of the questions in SECTION B. Each question in section A is worth 12 marks and each question in section B is worth 20 marks.

SECTION A

1. Differentiate the following functions

$$f(x) = x^3 - 2x^{-1/4} + \sqrt{x}, \qquad g(x) = \frac{2 - 3x^2}{x^2 - 4}, \qquad h(x) = \sin(2x)\cos(3x).$$

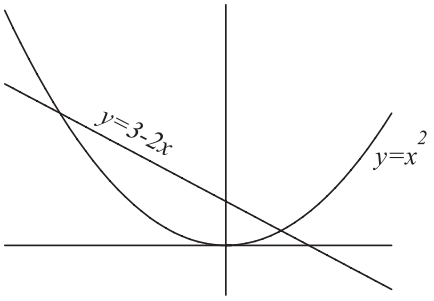
2. **(a)** Find the indefinite integrals

$$\int \left(x^{2/3} - 3e^{-x} + x^{-1}\right)\, dx, \qquad \int \left(\cos 3x + e^{3x+2}\right)\, dx.$$

(b) Evaluate the definite integrals

$$\int_0^1 \frac{x\, dx}{1 + x^2}, \qquad \int_0^{\pi/2} t \cos t\, dt.$$

3. **(a)** Find the coordinates of the points where the curve $y = x^2$ meets the line $y = 3 - 2x$.



Then find the area bounded by the curve and the line.

(b) Evaluate the definite integral $\displaystyle\int_0^{\pi/2} \sin^2\theta\, d\theta$ by using the identity $\cos 2\theta = 1 - 2\sin^2\theta$.

4. **(a)** Find the maximum and minimum values of the function $f(x) = x^3 + 3x^2 - 9x + 2$ on the interval $0 \leqslant x \leqslant 2$.

(b) Let $g(x) = x^2 e^{-x}$. For which positive x is the derivative of $g(x)$ strictly positive? Sketch the graph of $g(x)$ for $x \geqslant 0$ and find its maximum value for positive x.

5. Use the Newton-Raphson method to find the solution to the equation $e^x = 4 - x$ that is near $x = 1$. Start from $x = 1.0$ and state your result accurate to 3 decimal places.

SECTION B

6. **(a)** Show that $y = e^{-3x}\sin(2x + 1)$ satisfies the differential equation $y'' + 6y' + 13y = 0$.

(b) Find the second derivative of the function $\ln\left(\dfrac{1+x}{1-x}\right)$.

7. **(a)** Evaluate the definite integral $\displaystyle\int_2^3 \frac{x+2}{(2x+1)(x-1)}\,dx$.

(b) Solve the differential equations

$$\text{(i)} \quad \frac{dy}{dx} = 2x, \qquad \text{(ii)} \quad \frac{dy}{dx} = 2y.$$

Find the the solutions of the equations which satisfy $y = 4$ when $x = 1$.

8. **(a)** Find the equation of the tangent line to the curve $x^3 - y^3 - y = 1$ at the point $(1, 0)$.

(b) Define the inverse trigonometric function $\arcsin(x)$. Assuming that the derivative of $\sin(x)$ is $\cos(x)$, show that

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}.$$

9. An open-topped right circular cylinder of height h and radius r has to be constructed with a volume of 216π cubic metres. The cost of the metal used is £5 per square metre. Find the values of r and h which give the cheapest cylinder.