UNIVERSITY OF ABERDEEN **SESSION 2005–2006**

DEGREE EXAMINATION

ES 3006 Engineering Analysis and Methods 1B

Monday 23 January 2006 (9am-12 noon)

NOTES **(i) Candidates are permitted to use approved calculators.**

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

(iii) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Find a vector equation of the line through the points (1, 2, 0) and (4, 3, 1).

[3 marks]

(b) Let Γ be the curve given by $\mathbf{r} = (u^2 - 1, u^2 + u, 2u^3 - u)$. Find a unit tangent vector to Γ at the point $(0, 0, -1)$.

[5 marks]

(c) Let Σ be the surface given by $\mathbf{r} = (s + t,\ s + t^2,\ s - t)$. Find a non-zero vector which is normal to the surface at the point where $s = 1$ and $t = 2$.

[6 marks]

(d) Let P be a particle travelling in space. Suppose that the position vector $\mathbf{r}$ of P at time t is given by $\mathbf{r} = (\cos(3t),\ \sin(3t),\ 4t)$. Find the velocity, speed and acceleration of P when $t = \pi$. What distance does P travel in the time interval from time $t = 0$ to time $t = \pi$?

[6 marks]

2. **(a)** Find a coordinate equation of the tangent plane to the surface given by $y = x^2 + z^2$ at the point (1, 2, 1). Find a vector equation of the line normal to the surface at the same point.

[6 marks]

(b) Let ϕ be the scalar field given by $\phi = |\mathbf{r}|^3$ ($\mathbf{r} = (x,\ y,\ z)$). Show that

$$\operatorname{grad}\phi = 3\,|\mathbf{r}|\,\mathbf{r}.$$

[6 marks]

(c) Let ρ be a scalar field and let $\mathbf{v}$ be a vector field, both defined in space. Show that

$$\operatorname{div}(\rho\mathbf{v}) = (\operatorname{grad}\rho)\cdot\mathbf{v} + \rho\operatorname{div}\mathbf{v}.$$

Show also that

$$\operatorname{curl}(\operatorname{grad}\rho) = 0.$$

[8 marks]

3. **(a)** Let $\mathbf{F}$ be the vector field given by $\mathbf{F} = (yz,\ z(x+z),\ y(x+2z)\,)$. Is $\mathbf{F}$ a conservative (gradient) vector field? Justify your answer.

[4 marks]

(b) Let ρ be the scalar field given by $\rho = yz(x+y+z)$. Evaluate

$$\int_C \operatorname{grad}\rho \cdot d\mathbf{r}$$

where C is the curve given by $\mathbf{r} = (t, 1-t^2, t^3)$, $(1 \leqslant t \leqslant 2)$ oriented by the direction of increasing t.

[6 marks]

(c) Let T be the region in the xy-plane enclosed by the lines $y = 1$, $x = 0$ and $2y = x$. Sketch the region T. Evaluate

$$\iint_T xy^2\, dxdy.$$

[10 marks]

4. **(a)** Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (2xy + x^3,\ x^2 + 3x + y)$. Use Green's theorem to evaluate $\int_C \mathbf{v} \cdot d\mathbf{r}$, where C is the circle with centre at the origin, with radius 2 and with the positive orientation.

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic? Justify your answer.

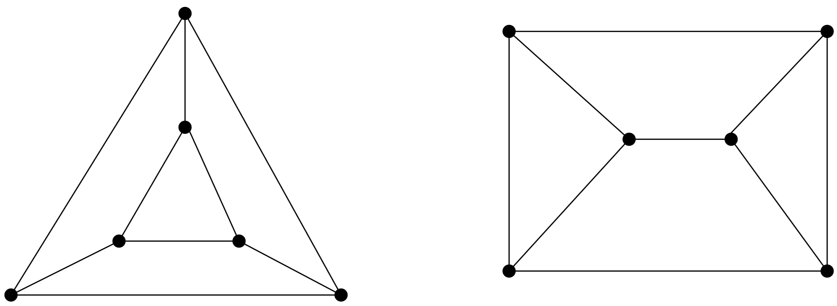

Figure 1 (Question 4 (b))

[10 marks]

/OVER

5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in weeks for the tasks of the project. Determine the minimum completion time, the float times of each node and a critical path.

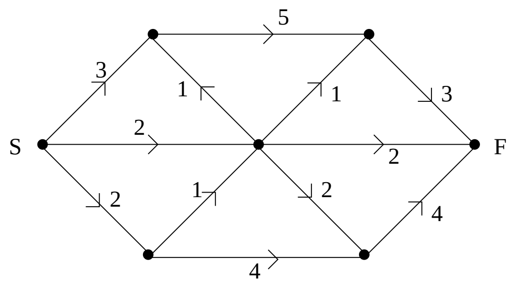

Figure 2 (Question 5 (a))

[10 marks]

(b) Find a maximal flow for the network represented by the diagram in Figure 3. Indicate in a diagram a maximal flow and a minimal cut of the network.

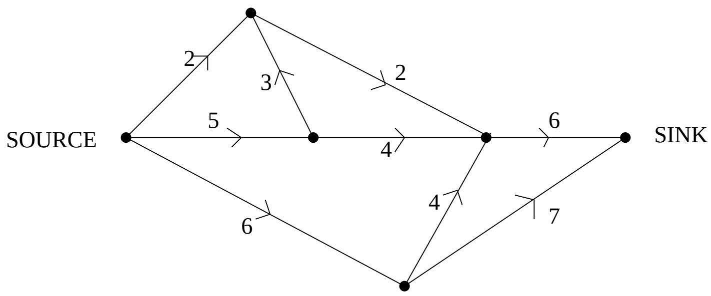

Figure 3 (Question 5 (b))

[10 marks]

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6. **(a)** Write MATLAB statements using `diff` and `subs` which will obtain an expression for the derivative dz/dy of

$$z = y^4 + 8y^3 - y + 1$$

and evaluate the derivative when $y = 2$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$\frac{dx}{dt} - 2x = \cos(t) \cdot e^t, \qquad \text{with } x = 1 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = -tx^3 \,.$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = -tx^3 \qquad \text{with } x = 1 \text{ when } t = 0,$$

in the range $0 \leq t \leq 2$.

[10 marks]

7. Consider the heat equation

$$\frac{\partial u}{\partial t} = 5\,\frac{\partial^2 u}{\partial x^2} \quad \text{for } -1 \leqslant x \leqslant 1 \text{ and } t \geqslant 0$$

subject to the (homogeneous) boundary conditions

$$u(t, -1) = 0, \quad u(t, 1) = 0 \quad \text{for } t \geqslant 0.$$

(i) Show that $u = e^{-5n^2\pi^2 t}\sin(n\pi x)$ satisfies the heat equation and the boundary conditions for any integer n.

Write down the solution of the heat equation which satisfies the boundary conditions and, in addition, the initial condition

$$u(0,\, x) = \sin(3\pi x) + 5\sin(5\pi x) \quad \text{for } -1 \leqslant x \leqslant 1.$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \leq t \leq 0.2$, $-1 \leq x \leq 1$, subject to the boundary conditions and, in addition, the initial condition

$$u(0, x) = x^2 - 1.$$

[12 marks]