UNIVERSITY OF ABERDEEN **SESSION 2004–2005**

Degree Examination

ES3006 Engineering Analysis and Methods 1B

Wednesday 26 January 2005 (2pm to 5pm)

NOTES (i) Candidates are permitted to use approved calculators.

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

(iii) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Find a vector equation of the line through the points (1, −1, 2) and (2, 3, 1).

[3 marks]

(b) Let Γ be the curve given by $\mathbf{r} = (s^2 + 1, s^2 - 1, s^3)$ (s a real variable). Find a unit tangent vector to Γ at the point (2, 0, −1).

[5 marks]

(c) Let Σ be the surface given by $\mathbf{r} = (u + v,\ u - v,\ u^2 + v)$. Find a normal to the surface at the point where $u = 1$ and $v = 2$.

[5 marks]

(d) Let P be a particle travelling in space. Suppose that the position vector $\mathbf{r}$ of P at time t is given by $\mathbf{r} = (3 \sin t,\ 3 \cos t,\ 4t)$. Find:

(i) the velocity of P at time t;
(ii) the acceleration of P at time t;
(iii) the magnitude of the velocity of P at time t.

[7 marks]

2. **(a)** Find a coordinate equation of the tangent plane to the surface given by $x = y^2 + z^2$ at the point (2, 1, 1). Find a vector equation of the line normal to the surface at the same point.

[8 marks]

(b) Let ρ be a scalar field and let $\mathbf{v}$ be a vector field, both defined in space. Show that

$$\operatorname{div}(\rho\mathbf{v}) = (\operatorname{grad}\rho) \cdot \mathbf{v} + \rho \operatorname{div}\mathbf{v}.$$

[6 marks]

(c) Let ϕ be a scalar field in space. Show that $\operatorname{curl}(\operatorname{grad}\phi) = \mathbf{0}$.

[6 marks]

3. **(a)** Let $\mathbf{F}$ be the vector field given by $\mathbf{F} = (yz,\ z(x+z),\ y(x+2z)\,)$. Show that the scalar field ϕ given by $\phi = yz(x+z)$ is a potential function for $\mathbf{F}$.

Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the line segment from (1, 0, 1) to (0, 1, 2).

[6 marks]

(b) Let $\mathbf{G} = (2xyz,\ xy^2z,\ xy^2z + z^2)$. Is $\mathbf{G}$ a conservative (gradient vector) field? Justify your answer.

[4 marks]

(c) Let T be the region in the xy-plane enclosed by the lines $y = 1$, $y = x$ and $x = 0$. Evaluate $\displaystyle\iint_T xy^2\,dxdy$.

[10 marks]

4. **(a)** Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (2xy + x^3,\ x^2 + 3x + y)$ Use Green's theorem to evaluate $\displaystyle\int_C \mathbf{v}\cdot d\mathbf{r}$ where C is the oriented curve consisting of the line segments from $(0,0)$ to $(1,1)$, from $(1,1)$ to $(0,1)$ and from $(0,1)$ to $(0,0)$.

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic? Justify your answer.

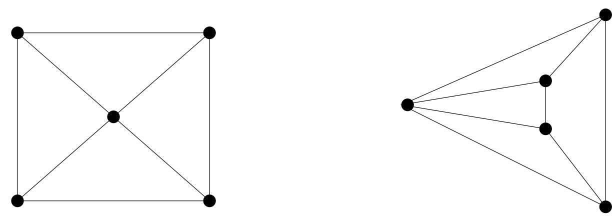

Figure 1 (Question 4 (b))

[10 marks]

/OVER

5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in days for the tasks of the project. Determine the minimum completion time, the float times of each node and a critical path.

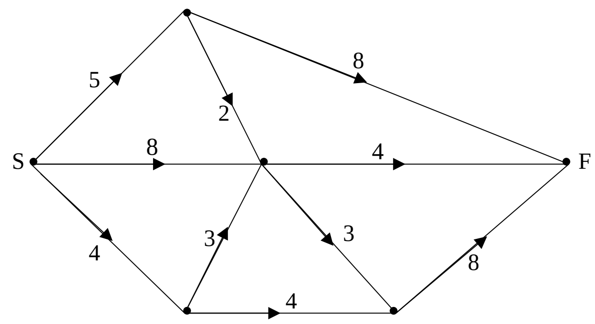

Figure 2 (Question 5 (a))

[10 marks]

(b) Find a maximal flow for the network represented by the diagram in Figure 3. Indicate in a diagram your maximal flow and a minimal cut of the network.

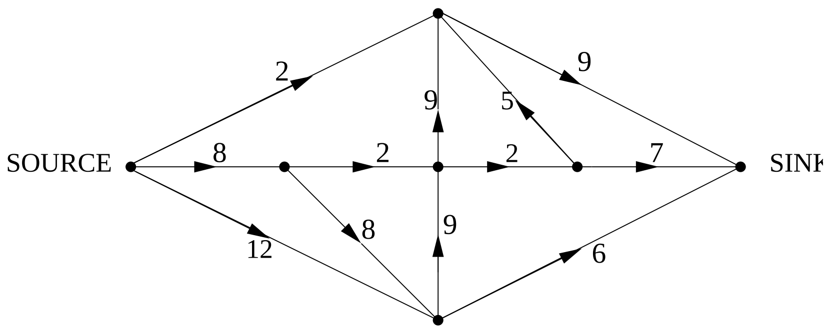

Figure 3 (Question 5 (b))

[10 marks]

/OVER

6. **(a)** Write MATLAB statements using `diff` and `subs` which will provide an expression for the derivative dy/dx of

$$y = x^3 + 5x^2 - x$$

and evaluate the derivative when $x = 2$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$\frac{dx}{dt} - 3x = 5\mathrm{e}^t, \qquad \text{with } x = 2 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = t - x^2 \,.$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = t - x^2 \qquad \text{with } x = 0 \text{ when } t = 0,$$

in the range $0 \le t \le 10$.

[10 marks]

7. Consider the heat equation

$$\frac{\partial u}{\partial t} = 4\,\frac{\partial^2 u}{\partial x^2}$$

for $-1 \leqslant x \leqslant 1$, subject to the (homogeneous) boundary conditions

$$u(t, -1) = 0, \quad u(t, 1) = 0 \quad \text{for } t \ge 0.$$

(i) Show that $u = e^{-4n^2\pi^2 t} \sin(n\pi x)$ satisfies the heat equation and the boundary conditions for any integer n.

Write down the solution of the heat equation which satifies the boundary conditions and, in addition, the initial condition

$$u(0, x) = 3\sin(3\pi x) + 4\sin(4\pi x).$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \le t \le 0.1$, $-1 \le x \le 1$, subject to the boundary conditions and, in addition, the initial condition

$$u(0, x) = \cos(3\pi x/2).$$

You should specify all the `function` M-files required.

[12 marks]