UNIVERSITY OF ABERDEEN **SESSION 2004–2005**

Degree Examination

ES3006 Engineering Analysis and Methods 1B

Monday 15 August 2005 (2 pm—5 pm)

NOTES **(i) Candidates are permitted to use approved calculators.**

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

(iii) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Find a coordinate equation for the plane through the points (2, 1, 0), (1, 2, 1) and (3, 4, 4).

[5 marks]

(b) Let Γ be the curve given by $\mathbf{r} = (s^3 - 1,\ s + 1,\ s^2 + 1)$. Find a tangent vector to Γ at the point where $s = 1$. Write down a vector equation of the tangent line to Γ at this point.

[5 marks]

(c) Let Σ be the surface given by $\mathbf{r} = (u + v,\ uv,\ u - v)$. Find a coordinate equation for the tangent plane to Σ at the point $(0, -1, -2)$.

[5 marks]

(d) Let P be a particle moving in space and suppose the position vector of P is given by $\mathbf{r} = (\cos(2t), \sin(2t), t)$ where t is the time from a fixed instant. Calculate the velocity of the particle and the magnitude of the acceleration of the particle.

[5 marks]

2. **(a)** Consider the surface given by $z + 2x^2 + y^2 = 4$. Find a vector normal to the surface at the point (1, 0, 2).

[4 marks]

(b) Let ϕ be a scalar field and $\mathbf{v}$ a vector field defined in space. Show that

$$\text{(i)}\quad \operatorname{div}(\phi\mathbf{v}) = (\operatorname{grad}\phi)\cdot\mathbf{v} + \phi\operatorname{div}\mathbf{v};$$
$$\text{(ii)}\quad \operatorname{curl}(\phi\mathbf{v}) = (\operatorname{grad}\phi)\times\mathbf{v} + \phi\operatorname{curl}\mathbf{v};$$

[11 marks]

(c) Let ϕ be a scalar field in space. Show that $\operatorname{curl}(\operatorname{grad}\phi) = \mathbf{0}$.

[5 marks]

3. **(a)** Let $\mathbf{F}$ be the vector field given by $\mathbf{F} = (2xy + z^2,\ 2yz + x^2,\ 2xz + y^2)$. Calculate $\operatorname{div}\mathbf{F}$. Verify that $\phi = x^2y + y^2z + z^2x$ is a potential function for $\mathbf{F}$.

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from (0, 0, 0) to (0, 1, 2).

[6 marks]

(b) Let $\mathbf{G} = (x^3,\ y^3,\ z^4)$. Is $\mathbf{G}$ a gradient vector field? Justify your answer.

[4 marks]

(c) Let D be the region in the xy-plane enclosed by the lines $x = 1$, $y = x$ and $y = 0$. Evaluate $\iint_D xy\,dxdy$.

[10 marks]

4. **(a)** Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (y, -x)$. Use Green's theorem to evaluate $\int_C \mathbf{v} \cdot d\mathbf{r}$ where C is the positively oriented circle of radius 2 and with centre at the point (3, 2).

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic? Justify your answer.

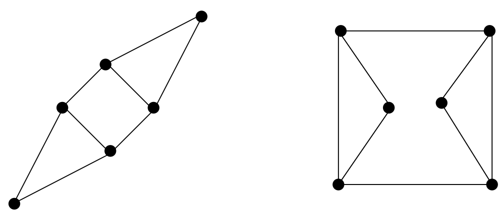

Figure 1 (Question 4 (b))

[10 marks]

/OVER

5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in days for the tasks of the project. Determine the minimum completion time, the float times of each node and a critical path.

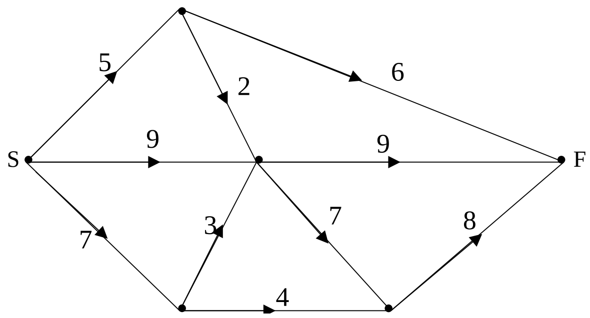

Figure 2 (Question 5 (a))

[10 marks]

(b) Find a maximal flow for the network represented by the diagram in Figure 3. Indicate in a diagram your maximal flow and a minimal cut of the network.

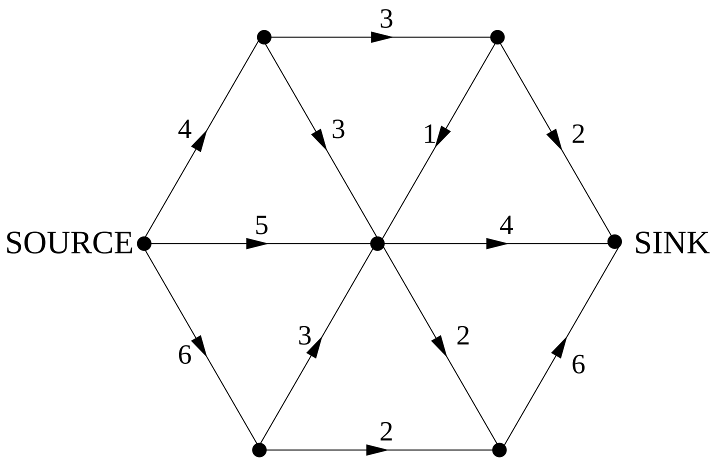

Figure 3 (Question 5 (b))

[10 marks]

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6. **(a)** Write MATLAB statements using `diff` and `subs` which will provide an expression for the derivative dy/dx of

$$y = 3e^{-x^2/2}$$

and evaluate the derivative when $x = 1$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$\frac{dx}{dt} + 2x = \cos(3t), \qquad \text{with } x = 5 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = tx^2\,.$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = tx^2 \qquad \text{with } x = 1 \text{ when } t = 0,$$

in the range $0 \le t \le 0.5$.

[10 marks]

7. Consider the heat equation

$$9\,\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $0 \le x \le 1$, subject to the (homogeneous) boundary conditions

$$\frac{\partial u}{\partial x}(t, 0) = 0, \quad \frac{\partial u}{\partial x}(t, 1) = 0 \quad \text{for } t \ge 0.$$

(i) Show that $u = e^{-\frac{n^2\pi^2 t}{9}} \cos(n\pi x)$ satisfies the heat equation and the boudary conditions for all integers n.

Write down the solution of the heat equation which satisfies the boundary conditions and, in addition, the initial condition

$$u(0, x) = 4 + 2\cos(2\pi x).$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \le t \le 1$, $0 \le x \le 1$, subject to the boundary conditions and, in addition, the initial condition

$$u(0, x) = 3\cos(\pi x).$$

You should specify all the `function` M-files required.

[12 marks]