UNIVERSITY OF ABERDEEN **SESSION 2006–2007**

Degree Examination

EG 3006 Engineering Analysis and Methods 1A

Tuesday 16 January 2007 (2 pm to 5 pm)

NOTES: (i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the examination, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook: `www.abdn.ac.uk/registry/quality/appendix7x1.pdf` **(Sections 4.14 and 5).**

(iv) Candidates are permitted to use approved calculators.

(v) Candidates are permitted to use the Engineering Mathematics Handbook, which will be provided for them.

(vi) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Let Γ be the curve given by $\mathbf{r} = (u^4 - 1,\, u^2 - 1,\, e^{u+1})$. Find a unit tangent vector to Γ at the point (0, 0, 1).

[4 marks]

(b) Let Σ be the surface given by $\mathbf{r} = (u+3v,\, 1+uv,\, v^2)$. Find a vector equation of the line normal to the surface Σ at the point (4, 2, 1). Find a coordinate equation of the tangent plane to the surface at the same point.

[8 marks]

(c) Let P be a particle moving in space and suppose that the position vector $\mathbf{r}$ of P relative to axes with origin O is given by

$$\mathbf{r} = (2t,\, 3t,\, t^{\frac{3}{2}})$$

where t is the time from a fixed instant. Calculate the velocity, speed and acceleration of P expressing your answers as functions of t. What distance does P travel in the time interval from $t = 0$ to $t = 1$?

[8 marks]

2. **(a)** Describe the direction and magnitude of the value of the gradient of a scalar field at a point in terms of properties of the field.

[3 marks]

(b) Find a coordinate equation for the tangent plane to the surface $x^2 + 4y^2 + z^2 = 6$ at the point (1, 1, 1).

[4 marks]

(c) Let $\mathbf{v}$ and $\mathbf{w}$ be a vector fields defined in space. Show that

$$\operatorname{div}(\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot \operatorname{curl} \mathbf{v} - \mathbf{v} \cdot \operatorname{curl} \mathbf{w}.$$

[6 marks]

(d) Let $\mathbf{G} = (3x^2y^3z + 2xyz^3,\ 3x^3y^2z + x^2z^3,\ x^3y^3 + 3x^2yz^2)$. It is given that $\mathbf{G}$ is a conservative vector field (you are *not* required to show this). Find a potential function for $\mathbf{G}$.

[7 marks]

3. **(a)** Show that the vector field $\mathbf{F} = (z - yx^4, x - zy^4,\ y - xz^4)$ is not a conservative vector field.

[4 marks]

(b) Let $\mathbf{v}$ be the vector field given by $\mathbf{v} = (y,\ z,\ x)$ and let Γ be the oriented curve from (0, 0, 0) to (1, 1, 1) parameterised by

$$\mathbf{r} = (t,\ t^2,\ t^3), \quad (0 \leqslant t \leqslant 1).$$

Evaluate $\displaystyle\int_\Gamma \mathbf{v} \cdot d\mathbf{r}$.

Let ρ be the scalar field given by $\rho = x^3y + 4xyz$. Evaluate $\displaystyle\int_\Gamma \operatorname{grad} \rho \cdot d\mathbf{r}$.

[8 marks]

(c) Let T be the triangular region in the xy-plane enclosed by the lines $y + x = 1$, $y = -1$ and $x = -1$. Evaluate $\displaystyle\iint_T (x + y)^2\, dx\, dy$.

[8 marks]

/OVER

4. **(a)** State Green's theorem.

Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (3x^2y + 6y + 1,\ x^3 + 4x + y^2)$.

Evaluate $\int_C \mathbf{v} \cdot d\mathbf{r}$ where C is the circle with the positive orientation, centred at the point (1, 1) and with radius 2.

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic ? Justify your answer.

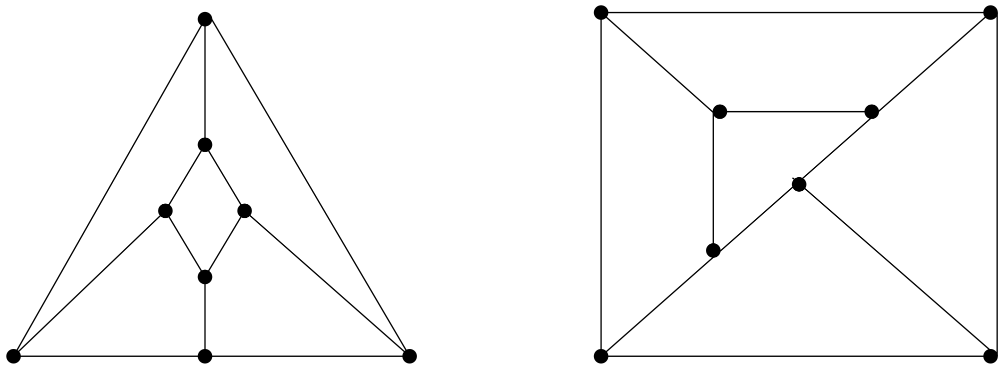

Figure 1 (Question 4 (b))

[10 marks]

/OVER

5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in days for the tasks of the project. Determine the minimum completion time, the float times of each vertex and a critical path.

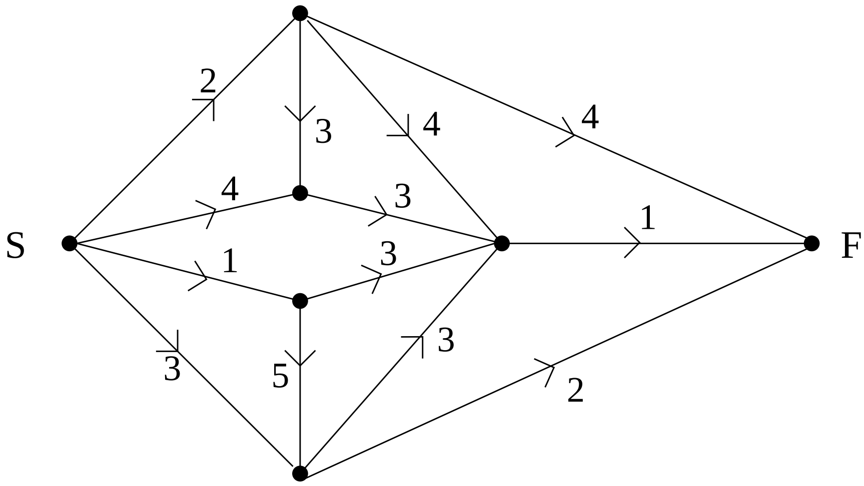

Figure 2 (Question 5 (a))

[10 marks]

(b) Find the value of a maximal flow for the network represented by Figure 3. Indicate in a diagram your maximal flow and a minimal cut.

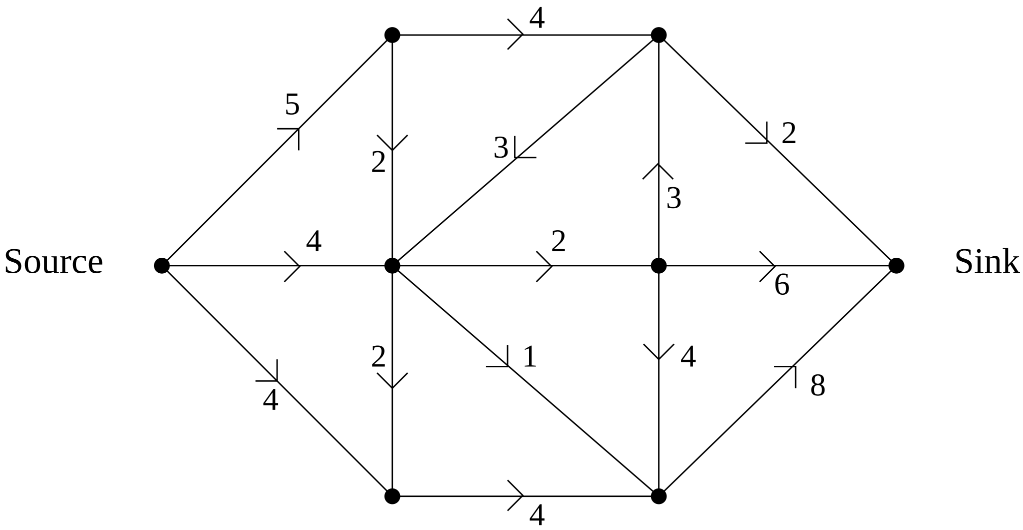

Figure 3 (Question 5 (b))

[10 marks]

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6. **(a)** Write MATLAB statements using `diff` and `subs` which will obtain an expression for the derivative dz/dt of

$$z = e^t \sin(t) - t^3$$

and evaluate the derivative when $t = 2$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$2\frac{d^2x}{dt^2} + \frac{dx}{dt} - 5x = \frac{1}{1+t}, \qquad \text{with } x = 1 \text{ and } \frac{dx}{dt} = 2 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = t^2 - x^2 \,.$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = t^2 - x^2 \qquad \text{with } x = 0 \text{ when } t = 0,$$

in the range $0 \leq t \leq 1$.

[10 marks]

7. Consider the heat equation

$$\frac{\partial u}{\partial t} = 3\,\frac{\partial^2 u}{\partial x^2} \quad \text{for } t \geqslant 0 \text{ and } -1 \leqslant x \leqslant 1.$$

(i) Find values of λ and μ so that $u = e^{\lambda t} \sin(\mu x)$ is a solution of the heat equation satisfying the homogeneous boundary conditions

$$u(t,\, -1) = 0, \qquad u(t,\, 1) = 0.$$

Write down the solution of the heat equation subject to the homogeneous boundary conditions and the initial condition

$$u(0,\, x) = 2\sin(\pi x) + 4\sin(3\pi x).$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \leqslant t \leqslant 0.2$ and $-1 \leqslant x \leqslant 1$, subject to the initial condition

$$u(0, x) = x + x^2$$

and boundary conditions

$$u(t, -1) = 0, \quad u(t, 1) = 2 \quad \text{for } t \geqslant 0.$$

You should specify all the `function` M-files required.

[12 marks]