UNIVERSITY OF ABERDEEN **SESSION 2005–2006**

Degree Examination

EG 3006 Engineering Analysis and Methods 1A

Monday 23 January 2006 (9am - 12 noon)

NOTES **(i) Candidates are permitted to use approved calculators.**

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

(iii) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Let Γ be the curve given by $\mathbf{r} = (u^2 + 1,\ u^3 + 2,\ u^2 + u^4)$. Find a vector equation of the line tangent to Γ at the point (2, 1, 2).

[5 marks]

(b) Let Σ be the surface given by $\mathbf{r} = (u(v + 1),\ u + v,\ v^2)$. Find a unit vector normal to the surface Σ at the point (0, 0, 1). Find a coordinate equation of the tangent plane to the surface at the same point.

[7 marks]

(c) Let P be a particle moving in space, let $\mathbf{r}$ be the position vector of P relative to an origin O and let $\mathbf{v}$ be the velocity of P ($\mathbf{r}$ and $\mathbf{v}$ depend on time t). By considering coordinates, or otherwise, show that

$$\frac{d}{dt}(\mathbf{r} \cdot \mathbf{r}) = 2\,\mathbf{r} \cdot \mathbf{v}.$$

Suppose that $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ where $\boldsymbol{\omega}$ is a constant vector. Assuming $\mathbf{r} \neq \mathbf{0}$, show that the particle is moving on a sphere with centre at the origin.

[8 marks]

2. **(a)** Describe the direction and magnitude of the value of the gradient of a scalar field at a point in terms of properties of the field.

[4 marks]

(b) Find a coordinate equation for the tangent plane to the surface $z = x^2 + y^2$ at the point (1, 1, 2).

[4 marks]

(c) Let $\mathbf{a}$ be a constant vector. A scalar field V is given by $V = |\mathbf{r} - \mathbf{a}|^5$, ($\mathbf{r} = (x,\ y,\ z)$). Show that

$$\operatorname{grad} V = 5\,|\mathbf{r} - \mathbf{a}|^3\,(\mathbf{r} - \mathbf{a}).$$

[5 marks]

(d) Let $\mathbf{v}$ and $\mathbf{w}$ be vector fields defined in space. Show, carefully, that

$$\operatorname{div}(\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot \operatorname{curl} \mathbf{v} - \mathbf{v} \cdot \operatorname{curl} \mathbf{w}.$$

[7 marks]

3. **(a)** Let $\mathbf{F}$ be the vector field given by $\mathbf{F} = (xyz,\ xy,\ x + y + z)$. Is $\mathbf{F}$ a conservative vector field? Justify your answer. (You are *not* required to find a potential if it is conservative.)

[5 marks]

(b) Let $\mathbf{G}$ be the vector field given by $\mathbf{G} = (3x^2y + 2xyz,\ x^3 + x^2z + z,\ x^2y + y)$. It is given that $\mathbf{G}$ is a conservative vector field (you are *not* required to show this). Find a potential function for $\mathbf{G}$.

Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$ where C is the line segment from $(0,\ 0,\ 0)$ to $(1,\ -1,\ 2)$).

[8 marks]

(c) Evaluate $\iint_T xy^2\, dx\, dy$ where T is the triangular region in the xy-plane enclosed by the lines $2y + x = 0$, $y = 1$ and $x = 0$.

[7 marks]

4. **(a)** Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (3x^2y + 3y + 1,\ x^3 + 4x + y)$. Use Green's theorem to evaluate $\int_C \mathbf{v} \cdot d\mathbf{r}$ where C is the oriented curve consisting of the line segments from $(0,\ 0)$ to $(0,\ 2)$, from $(0,\ 2)$ to $(2,\ 0)$ and from $(2,\ 0)$ to $(0, 0)$.

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic? Justify your answer.

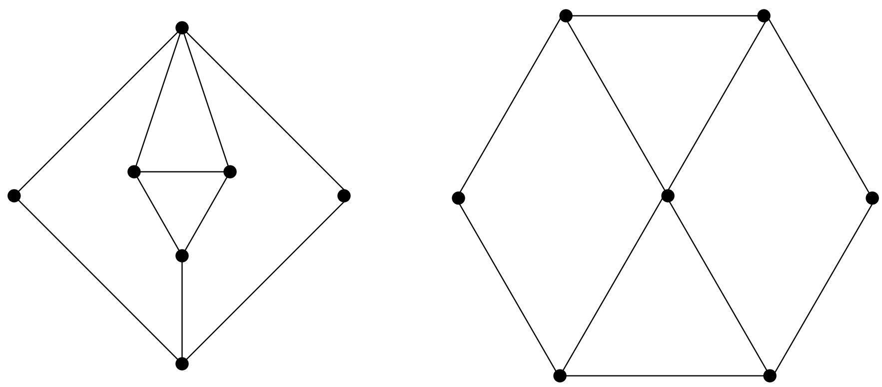

Figure 1 (Question 4 (b))

[10 marks]

/OVER

5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in days for the tasks of the project. Determine the minimum completion time, the float times of each vertex and a critical path.

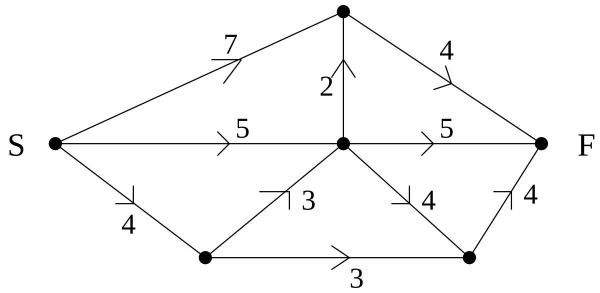

Figure 2 (Question 5 (a))

[10 marks]

(b) Find the value of a maximal flow for the network represented by Figure 3. Indicate in a diagram a maximal flow and a minimal cut.

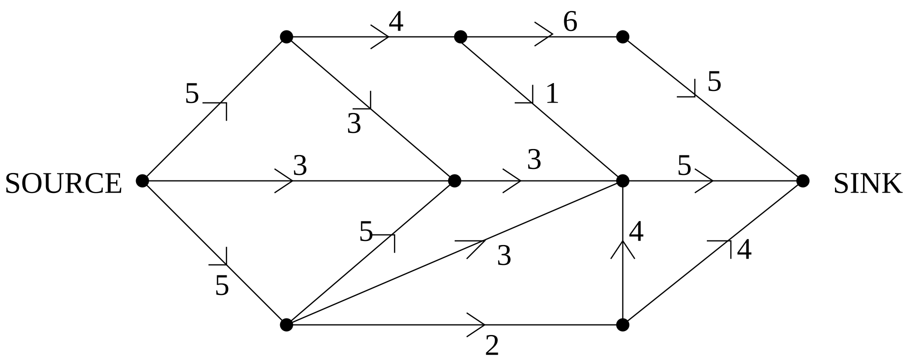

Figure 3 (Question 5 (b))

[10 marks]

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6. **(a)** Write MATLAB statements using `diff` and `subs` which will obtain an expression for the derivative dz/dy of

$$z = y^4 + 8y^3 - y + 1$$

and evaluate the derivative when $y = 2$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = e^{-2t}, \qquad \text{with } x = 4 \text{ and } \frac{dx}{dt} = 3 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = -tx^2 .$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = -tx^2 \qquad \text{with } x = 1 \text{ when } t = 0,$$

in the range $0 \leq t \leq 2$.

[10 marks]

7. Consider the heat equation

$$\frac{\partial u}{\partial t} = 2\,\frac{\partial^2 u}{\partial x^2} \quad \text{for } -2 \leqslant x \leqslant 2 \text{ and } t \geqslant 0.$$

(i) Find the values of λ and μ so that $u = e^{\lambda t}\sin(\mu x)$ is a solution of the heat equation subject to the *homogeneous* boundary conditions

$$u(t,\,-2) = 0, \quad u(t,\,2) = 0 \quad \text{for } t \geqslant 0.$$

Write down the solution of the heat equation subject to the homogeneous boundary conditions and to the initial condition

$$u(0,\,x) = \sin(\pi x) + 2\sin(2\pi x).$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \leqslant t \leqslant 0.2$, $-2 \leqslant x \leqslant 2$ subject to the *non-homogeneous* boundary conditions

$$u(t, -2) = 0, \quad u(t, 2) = 2 \quad \text{for } t \geqslant 0$$

and to the initial condition

$$u(0, x) = \begin{cases} 0 & \text{if } x < 0, \\ x & \text{if } x \geq 0. \end{cases}$$

You should specify all the `function` M-files required.

[12 marks]