UNIVERSITY OF ABERDEEN **SESSION 2004–2005**

DEGREE EXAMINATION

EG3006 Engineering Analysis and Methods 1A

Wednesday 26 January 2005 (2pm to 5pm)

NOTES (i) Candidates are permitted to use approved calculators.

(ii) Candidates are permitted to use the Engineering Mathematics Handbook which will be provided for them.

(iii) The specification of some MATLAB commands is appended for use in questions 6 and 7.

Candidates should attempt FIVE questions. All questions carry 20 marks.

1. **(a)** Let Γ be the curve given by $\mathbf{r} = (u^2 + 1,\ u^3 + 2,\ 2u^2 + u^4)$. Find a vector equation of the tangent line to Γ at the point (2, 1, 3).

[5 marks]

(b) Let Σ be the surface given by $\mathbf{r} = (u + v,\ u(v + 1),\ v^2)$. Find a unit vector normal to the surface Σ at the point (0, 0, 1).

[7 marks]

(c) Let P be a particle moving in a circular orbit in the xy-plane. The position vector of P (relative to O) is given by $\mathbf{r} = (R\cos(\omega t),\ R\sin(\omega t))$ where R and ω are constants and t is time from a fixed instant. Show that the acceleration of P is $-\omega^2\,\mathbf{r}$.

The orbit of a satellite about the Earth is a circle of radius 4.21×10^7 metres. The satellite makes one complete revolution in exactly 24 hours. Assuming the angular speed about the centre of the orbit is constant, find the magnitude of the acceleration of the satellite (expressed in metres per second per second).

[8 marks]

2. **(a)** Describe the direction and magnitude of the value of the gradient of a scalar field at a point in terms of properties of the field.

[4 marks]

(b) Find a coordinate equation for the tangent plane to the surface $y = x^2 + z^2$ at the point (1, 2, 1).

[4 marks]

(c) A vector field $\mathbf{F}$ is given by $\mathbf{F} = \operatorname{grad} V$ where $V = |\mathbf{r}|^3$, $(\mathbf{r} = (x,\ y,\ z)\,)$. Show that

$$\mathbf{F} = 3\,|\mathbf{r}|\,\mathbf{r}.$$

[6 marks]

(d) Let ϕ be a scalar field and let $\mathbf{v}$ be a vector field, both defined in space. Show, carefully, that

$$\operatorname{curl}(\phi\mathbf{v}) = \phi \operatorname{curl}\mathbf{v} + (\operatorname{grad}\phi) \times \mathbf{v}.$$

[6 marks]

3. **(a)** Let $\mathbf{F}$ be the vector field given by $\mathbf{F} = (x^2 + yz,\ y + xz,\ z + xyz)$. Is $\mathbf{F}$ a conservative vector field? (You are *not* required to find a potential if it is conservative.)

[5 marks]

(b) Let $\mathbf{G}$ be the vector field given by $\mathbf{G} = (2zx + y^2,\ 2xy + z^2,\ 2yz + x^2)$. It is given that $\mathbf{G}$ is conservative (you are *not* required to show this). Find a potential function for $\mathbf{G}$.

Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$ where C is the line segment from $(1,\ 2,\ 1)$ to $(3,\ 1,\ 2)$.

[8 marks]

(c) Evaluate $\iint_T yx^2\,dx\,dy$ where T is the triangular region in the xy-plane enclosed by the lines $y = x$, $y = -x$ and $y = 1$.

[7 marks]

4. **(a)** Let $\mathbf{v}$ be the vector field in the xy-plane given by $\mathbf{v} = (2xy + x^3,\ x^2 + 3x + y)$. Use Green's theorem to evaluate $\int_C \mathbf{v} \cdot d\mathbf{r}$ where C is the oriented curve consisting of the line segments from $(0,0)$ to $(0,1)$, from $(0,1)$ to $(1,1)$ and from $(1,1)$ to $(0,0)$.

[10 marks]

(b) Are the two graphs, represented by the diagrams in Figure 1, isomorphic? Justify your answer.

Figure 1 (Question 4 (b))

[10 marks]

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5. **(a)** The scheduling network for a construction project is shown in Figure 2. The weights are the completion times in days for the tasks of the project. Determine the minimum completion time, the float times of each vertex and a critical path.

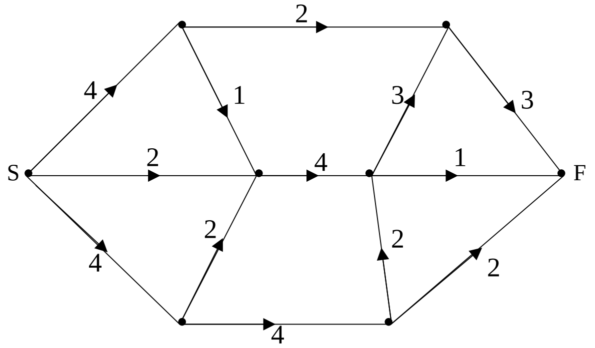

Figure 2 (Question 5 (a))

[10 marks]

(b) The diagram in Figure 3 represents a network. The capacities on the network are indicated by the numbers which are *not* circled. The numbers which are circled indicate a flow on the network. Indicate on a diagram of the network a path joining the source and sink which has spare capacity. Find a maximal flow for the network. Indicate on the diagram a maximal flow and a minimal cut of the network.

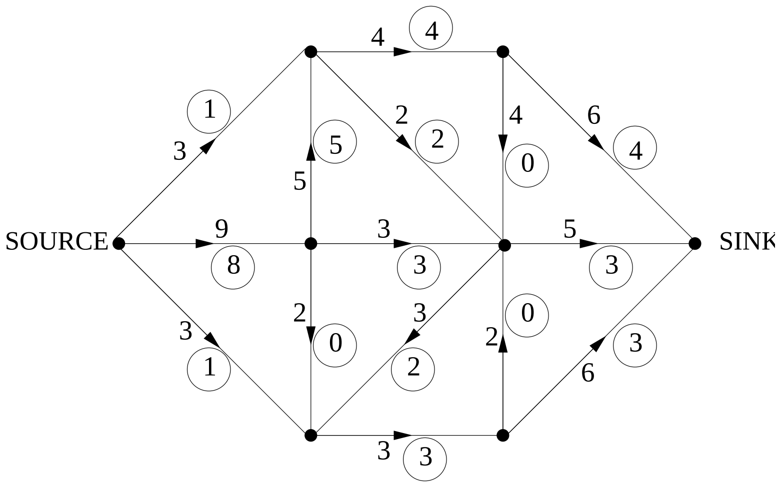

Figure 3 (Question 5 (b))

[10 marks]

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6. **(a)** Write MATLAB statements using `diff` and `subs` which will provide an expression for the derivative dy/dx of

$$y = x^3 + 5x^2 - x$$

and evaluate the derivative when $x = 2$.

[5 marks]

(b) Write MATLAB statements involving `dsolve` which will give an exact (analytic) solution of the initial value problem:

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = \sin(2t), \qquad \text{with } x = 5 \text{ and } \frac{dx}{dt} = -2 \text{ when } t = 0.$$

[5 marks]

(c) Write a MATLAB `function` M-file `f.m` to define the function

$$f(t, x) = t - x^2\,.$$

Using the routine `ode45`, give a MATLAB statement that produces a numerical solution of the initial value problem

$$\frac{dx}{dt} = t - x^2 \qquad \text{with } x = 0 \text{ when } t = 0,$$

in the range $0 \le t \le 10$.

[10 marks]

7. Consider the heat equation

$$\frac{\partial u}{\partial t} = 4\,\frac{\partial^2 u}{\partial x^2}$$

for $-1 \le x \le 1$, subject to the (homogeneous) boundary conditions

$$u(t, -1) = 0, \quad u(t, 1) = 0 \quad \text{for } t \ge 0.$$

(i) Find the values of λ and μ so that $u = \mathrm{e}^{\lambda t} \sin(\mu x)$ is a solution of the heat equation satisfying the boundary conditions.

Write down the solution of the heat equation which satisfies the boundary conditions and, in addition, the initial condition

$$u(0, x) = 2\sin(\pi x) + 3\sin(2\pi x).$$

[8 marks]

(ii) Provide MATLAB code using the routine `pdepe` to obtain a numerical solution of the heat equation for $0 \le t \le 0.1$, $-1 \le x \le 1$, subject to the boundary conditions and, in addition, the initial condition

$$u(0, x) = \begin{cases} 1 + x & \text{if } x < 0, \\ 1 - x & \text{if } x \ge 0. \end{cases}$$

You should specify all the `function` M-files required.

[12 marks]